Week 5/6
Term 3
2023

HAWKER COLLEGE
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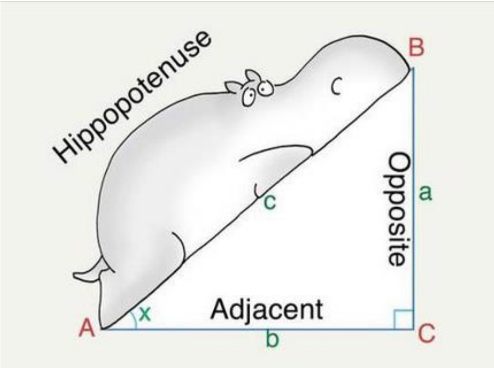
MA2

Applications of Trigonometry

Goals



This fortnight we are going to:

- review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle
- solve practical problems involving the trigonometry of right-angled including problems involving angles of elevation and depression and the use of bearings in navigation

Theoretical Components

Resources:
PDF file: Week 5/6 Notes and Exercises
Trigonometric ratios
https://www.youtube.com/watch?v=KQhQSd7Wigo
Angles of elevation and depression
https://www.youtube.com/watch?v=Sja5rEqmpa4

Knowledge Checklist
- Labelling sides of right-angled triangles
- Sine, cosine and tangent ratio
- Finding sides and angles of triangles
- Angles of elevation and depression
- Bearings
- Angles

Order
1. Read through the notes and examples
2. Work through the exercises
3. Complete the investigation at the end of the booklet.
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Investigation

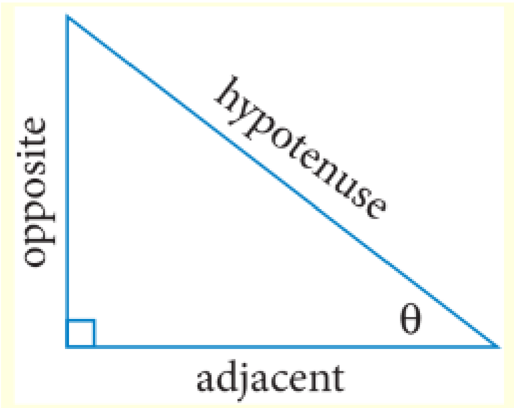
Complete the task at the end of the booklet and submit your work for checking. 😊

QFO

In **Week 7** you are to sit an **In-Class Task** worth 20% (with your weekly investigations) in your first lesson for the week (Monday). It is an **“open book”** task given under test conditions. You are allowed to bring in any of your notes and worked exercises since Week 1 and, of course, your calculator. You will need to come prepared with your Chromebook/laptop.

The ancient Greeks, around 100 BC, laid the foundations of a new branch of mathematics, one that uses angles, triangles and circles to calculate lengths and distances that cannot be measured physically. This new mathematics is now called **trigonometry**, from the Greek words *trigon* and *metron*, meaning 'triangle' and 'measure' respectively. Trigonometry is used widely today to calculate immeasurable lengths and distances—in engineering, surveying, navigation, astronomy, electronics and construction.

Trigonometry initially used right-angled triangles.



SOH	CAH	TOA
$\sin\theta = \dfrac{opposite}{hypotenuse}$	$\cos\theta = \dfrac{adjacent}{hypotenuse}$	$\tan\theta = \dfrac{opposite}{adjacent}$
$\sin\theta = \dfrac{O}{H}$	$\cos\theta = \dfrac{A}{H}$	$\tan\theta = \dfrac{O}{A}$
Finding the angle		
$\theta = \sin^{-1}\left(\dfrac{O}{H}\right)$	$\theta = cos^{-1}\left(\dfrac{A}{H}\right)$	$\theta = tan^{-1}\left(\dfrac{O}{A}\right)$
Finding the side		
$O = H \times \sin\theta$	$A = H \times \cos\theta$	$O = A \times \tan\theta$
$H = \dfrac{O}{\sin\theta}$	$H = \dfrac{A}{\cos\theta}$	$A = \dfrac{O}{\tan\theta}$

DEFINING THE TRIGONOMETRIC RATIOS - SOH CAH TOA

Trigonometry is the mathematics of using angles and triangles to calculate lengths and distances that are either difficult or impossible to measure. It uses the three sides of a right-angled triangle, each of which has a special name related to a particular angle in the triangle:

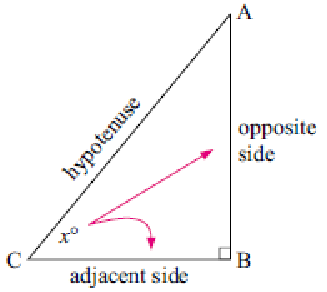
In a right-angled triangle, the side opposite the right-angle is called the **hypotenuse**.

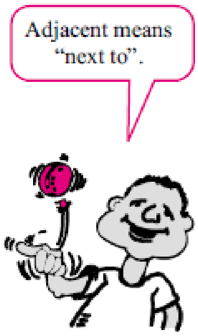
For further work, in trigonometry, the other sides are given names in relation to the other angles of the triangle as shown:

Relative to the angle marked x^o:

AB is the side which is opposite the angle. It is called the **opposite side**.

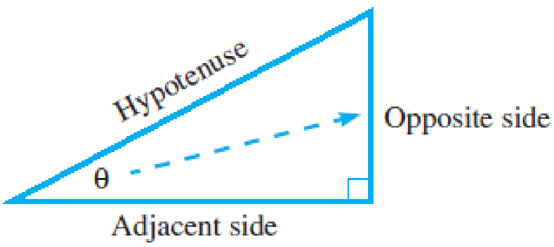
CB is the side which is adjacent to the angle. It is called the **adjacent side**.





- **Opposite side:** the side directly opposite the angle, not joined to the angle
- **Adjacent side:** the side leading to the right angle ('adjacent' means next to)
- **Hypotenuse:** the longest side (this has already been introduced with Pythagoras' theorem).

The diagram shows the three sides related to angle θ in the triangle.



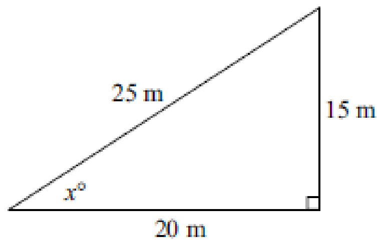
In trigonometry, three ratios are used: **sine**, **cosine**, and **tangent**. These are abbreviated to **tan θ**, **sin θ and cos θ**

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\quad\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ and $\quad\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

Make sure you label the three sides first before deciding which trigonometric ratio to use.

Example

Calculate the value of $\sin x^o$, $\cos x^o$ and $\tan x^o$ in the triangle alongside, which is not drawn to scale.



$$\sin x^o = \frac{\text{O}}{\text{H}} = \tfrac{15}{25} = \tfrac{3}{5} \text{ or } 0.6$$

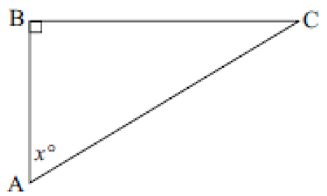
$$\cos x^o = \frac{\text{A}}{\text{H}} = \tfrac{20}{25} = \tfrac{4}{5} \text{ or } 0.8$$

$$\tan x^o = \frac{\text{O}}{\text{A}} = \tfrac{15}{20} = \tfrac{3}{4} \text{ or } 0.75$$

EXERCISE 1

1. Write the names of the sides (opposite, adjacent or hypotenuse) with respect to the reference angle on each side.

a)



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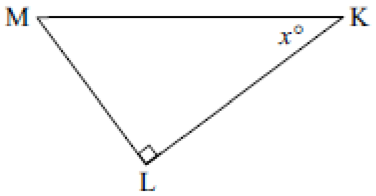

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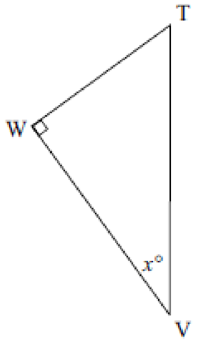

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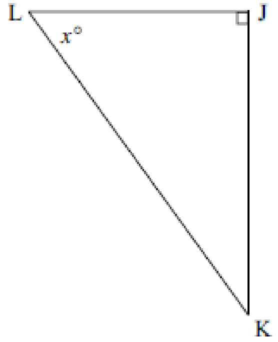
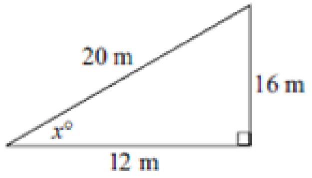


2. Write down the value of sin x, cos x <u>and</u> tan x ratios in the triangles below, which are not drawn to scale.

a)



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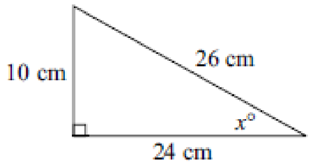

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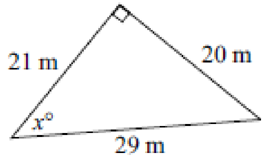

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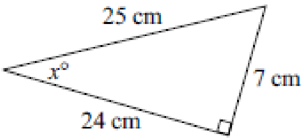
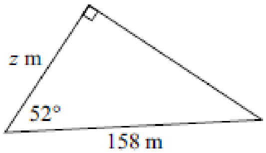


FINDING SIDES

Example

Find the length of the unknown side in this triangle.



Solution

In this triangle the hypotenuse is 158 m

z is next to the angle $52°$ and is thus the adjacent side.

Knowing the hypotenuse and the adjacent side means we use *cosine ratio.*

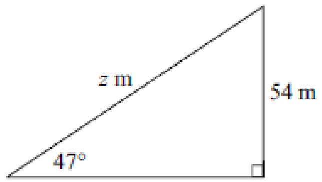
$$cos\ 52° = \frac{O}{H} = \frac{z}{158}$$

Thus, $z = 158 \times cos\ 52°$

$z = 97.3\ m$

Example

Here we have the adjacent and opposite sides, thus we will use *sin.*



$\sin 47^\circ = \frac{O}{H}$

$\sin 47^\circ = \frac{54}{z}$

Here the unknown side is the <u>denominator</u> and in this case we solve by dividing.

$z = \frac{54}{\sin 47}$ which gives $z = \frac{54}{\sin 47}$ Thus $z = 73.9$ m

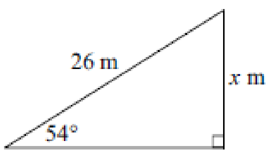
EXERCISE 2

In the following questions

- identify the sides as hyp, opp and adj
- decide which ratio you need to use
- carry out the working as shown in the example

1. Find the length of the unknown side to one decimal place.

a)



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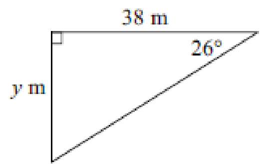

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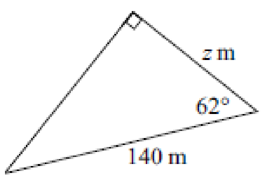

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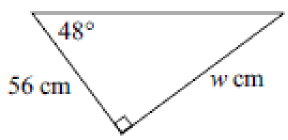
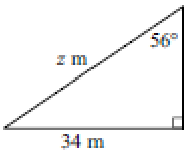


2. Find the length of the unknown side.

a)



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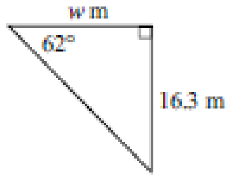


FINDING ANGLES

Trigonometric Ratios by Calculator

The *sin*, *cos* and *tan* ratios can be found by using a calculator.

Example

Using the *sin*, *cos* or *tan* button on a calculator gives the following.

$\sin 48^\circ = 0.743$ $\cos 53^\circ = 0.602$ $\tan 5^\circ = 0.087$

Using a calculator to find an angle

An angle can be found if we know the ratio.

Example

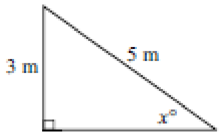
Find the size of the angle x° if $\sin x^\circ = 0.64$

Enter 0.64 Press $\boxed{\text{SHIFT}}$ $\boxed{\text{sin}}$ or press $\boxed{\text{SHIFT}}$ $\boxed{\text{sin}}$ 0.64 $\boxed{=}$

The calculator displays 39.7918… ie $x^\circ = 39.8^\circ$ Check the sequence on your calculator.

Example

Find the size of angle x° in the fiqure shown.



The two sides given are O and H, thus we need to use *sin.*

$$\sin x^\circ = \frac{O}{H} = \frac{3}{5} = 0.6$$

Now using "shift sin" 0.6 then = gives an angle of 36.9°

EXERCISE 3

1. Use a calculator to find, correct to 3 decimal places, the value of:

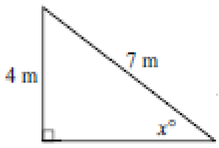
a) sin 47° b) cos 18° c) tan 86°

2. Find the size of the angle, x°, to the nearest degree if;

a) $\sin x^\circ = 0.7$ b) $\cos x^\circ = 0.4$ c) $\tan x^\circ = 1.25$

3. Find the value of the unknown angle to the nearest degree:

a)



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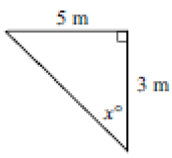

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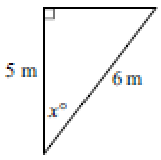

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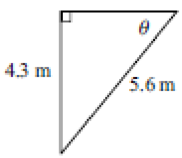
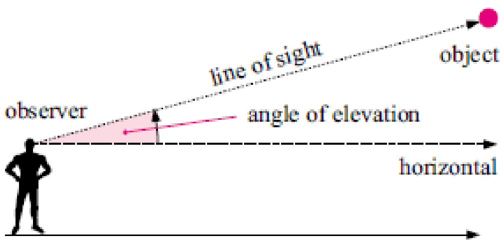
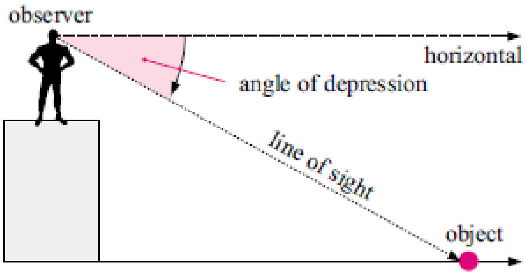


ANGLES OF ELEVATION AND DEPRESSION

The **angle of elevation** of an object, from an observer, is the angle between the horizontal and the line of sight **up** to the object.

The **angle of depression** of an object, from an observer, is the angle between the horizontal and the line of sight **down** to the object.





Example

The angle of depression from the top of a cliff 84 m above sea level, to a ship, is 22^o. Find the distance of the ship from the base of the cliff.

Solution

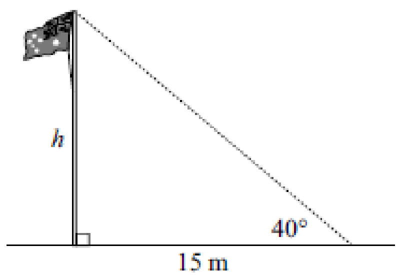
The angle of elevation of the top of a flagpole, as observed from a point 15 m from its base, is 40^o. Find the height of the flagpole.

Draw a diagram and let the height of the pole be h.

By trigonometry, $\dfrac{h}{15} = \tan 40^o = 0.839....$

$\therefore \quad h = 15 \times 0.839....$

$\therefore \quad h = 12.6$ m to 1 d.p.



Example

Draw a diagram and let the distance be d.

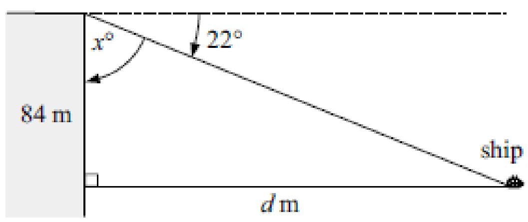
In the diagram, the angle $x^o = 90^o - 22^o$

i.e., $x^o = 68^o$

By trigonometry, $\dfrac{d}{84} = \tan 68^o = 2.475....$

Hence, $d = 84 \times 2.475....$

$\therefore \quad d = 208$ m to the nearest metre.



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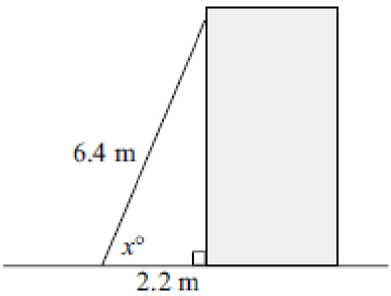
EXERCISE 4

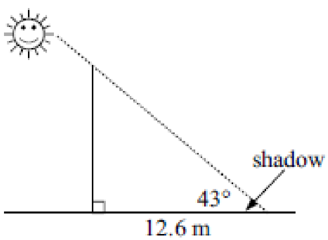
In these questions use the following steps.

- If there is no diagram, draw one and label the sides.
- Identify the ratio to be used
- Then solve in the usual way

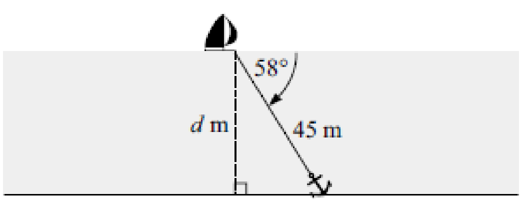
1. The foot of a ladder 6.4 m long is placed 2.2 m from the base of a wall. Calculate the angle the ladder makes with the ground, to the nearest degree.



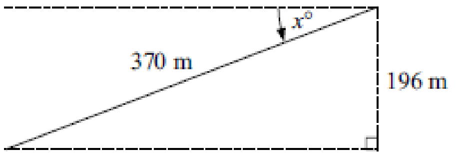
2. A flagpole casts a shadow 12.6 m long when the angle of elevation of the sun is 43°. Find the height of the flagpole.



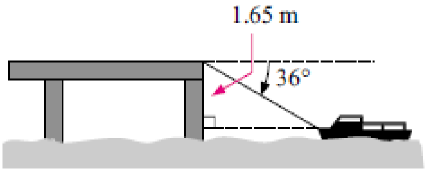
3. The anchor rope of a boat is 45 m long. When it is let out fully, it makes an angle of 58° with the surface of the water. Calculate the depth of the water at this point.



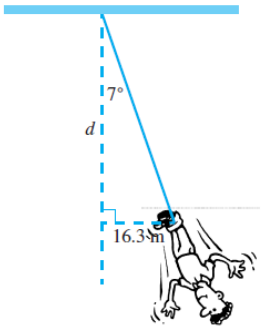
4. A ski slope falls 196 m over a run of 370 m. Find the angle the slope makes with the horizontal.



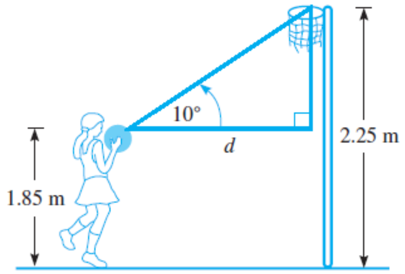
5. A boat is tied to a wharf which is 1.65 m above the boat, as shown in the diagram. The rope makes an angle of 36° with the horizontal. How long is the rope?



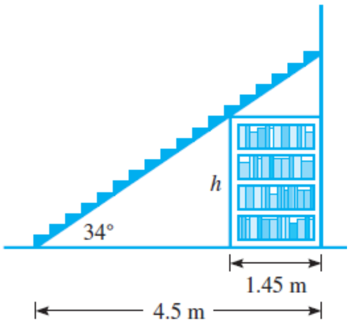
6. A bungy jumper leaps off at an angle of 7° to the vertical. If he is swinging 16.3 m off-centre, what vertical distance has he dropped (correct to 1 decimal place)?



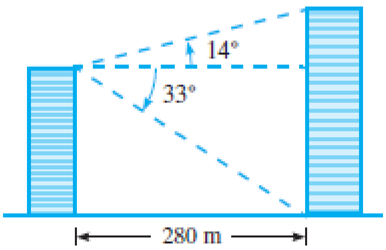
7. What is the distance between the netballer and the post if her shooting angle is 10°?



8. A stairwell is inclined at 34° to the ground floor and has a horizontal length of 4.5 m. Ronnie wants to place a bookcase of length 1.45 m underneath the stairwell. What is the height *h* of the tallest bookcase that can fit under the stairwell? Write your answer correct to 2 decimal places.



9. Emily stands at the top of her city apartment block and observes a taller office tower 280 m away. The angle of elevation of the top of the tower is 14° and the angle of depression is 33°. Calculate the height of the tower to the nearest metre. Set your working out carefully and write your answer correct to 2 decimal places.



10. From a point 45 m from the base of a tree, Julie observed an angle of elevation of 32° to the top of the tree. Calculate the height of the tree correct to 1 decimal place (draw a diagram).

11. There is lightning rod on the top of a building. From a location 50 metres from the base of the building, the angle of elevation to the top of the building is measured to be 36°. From the same location, the angle of elevation to the top of the lightning rod is measured to be 38°. Find the height of the lightning rod.

12. Lisa stands near a flagpole of height 4.5 m. The flagpole has a shadow of length 8 m while Lisa's shadow is 3 m. Find:

a) θ, the angle of elevation of the Sun, to the nearest degree.

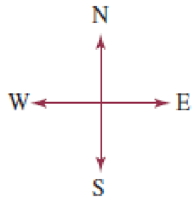
b) Lisa's height to the nearest centimetre using the value of θ found in part (a)

BEARINGS

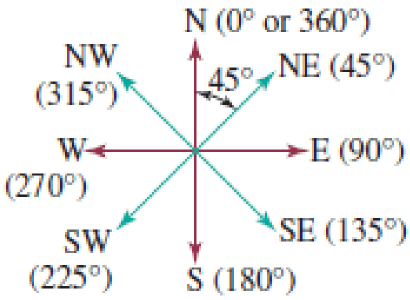
Bearings are used to locate the position of objects or the direction of a journey on a two-dimensional horizontal plane. Bearings or directions are straight lines from one point to another. To find bearings, a compass rose (a diagram, as shown below left, showing N, S, E and W) should be drawn centred on the point from where the bearing measurement is taken.

There are three main ways of specifying bearings or direction:

1. standard compass bearings (for example, N, SW, NE)
2. other compass bearings (for example, N10°W, S30°E, N45°37′E)
3. true bearings (for example, 100°T, 297°T, 045°T, 056°T)



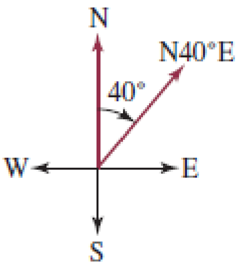
Standard compass bearings
There are 8 main standard bearings as shown in the diagrams below. The N, S, E and W standard bearings are called *cardinal points*.



It is important to consider the angles between any two bearings. For example, the angles from north (N) to all 8 bearings are shown in brackets in the diagrams above.

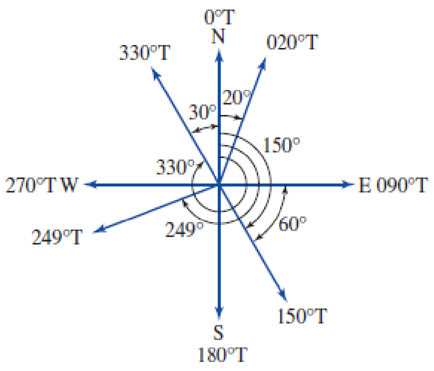
Other compass bearings
Often the direction required is not one of the 8 standard bearings.
To specify other bearings the following approach is taken.

1. Start from north (N) or south (S).
2. Turn through the angle specified towards east (E) or west (W).



True bearings

True bearings is another method for specifying directions and is commonly used in navigation.



To specify true bearings, first consider the following:

1. the angle is measured from north
2. the angle is measured in a clockwise direction to the bearing line
3. the angle of rotation may take any value from 0° to 360°
4. the symbol T is used to indicate it is a true bearing, for example, 125°T, 270°T
5. for bearings less than 100°T, use three digits with the first digit being a zero to indicate it is a bearing, for example, 045°T, 078°T.

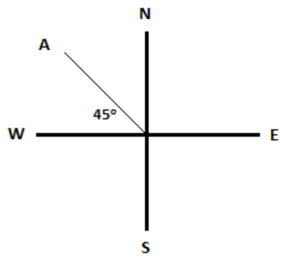
Example

Specify the direction in the figure at right as:

a) a standard compass bearing

b) a compass bearing

c) a true bearing.



Solution

a) The bearing is half-way between N and W, so compass bearing is NW

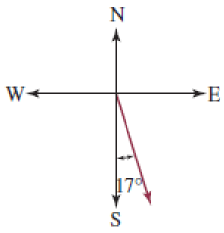
b) The compass bearing id N45̊W

c) West is at 270̊ and 270 + 45 = 315 so the true bearing is 315°T

Example

Draw a suitable diagram to represent the following directions.

a) S17°E



Draw the 4 main standard bearings. A compass bearing of S17°E means start from south; turn 17° towards east. Draw a bearing line at 17°. Mark in an angle of 17°.

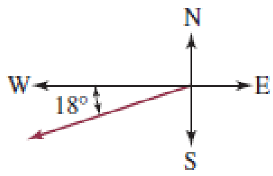
b) 252°T

A true bearing of 252°T is more than 180° and less than 270°, difference from west = 270° - 252° = 18°

The direction lies between south and west.

Find the difference between the bearing and west (or south).

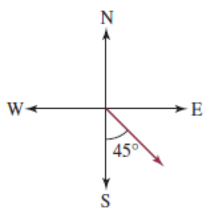
Draw the 4 main standard bearings and add the bearing line. Add the angle from west (or south).



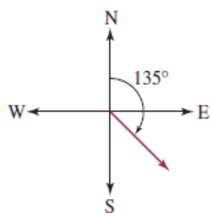
EXERCISE 5

1. Specify the following directions as standard compass bearings.

a)



b)



2. Convert the following true bearings to compass bearings.

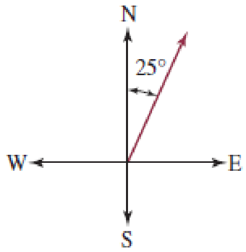
a) 040°T

b) 120°T

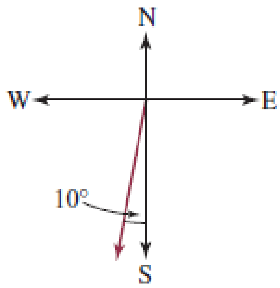
c) 350°T

3. Specify the following directions as compass bearings and true bearings.

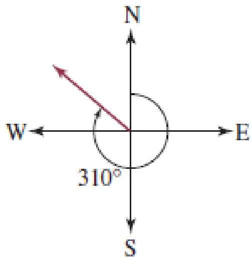
a)



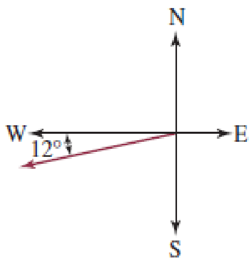
b)



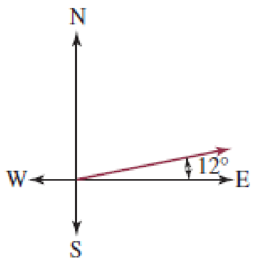
c)



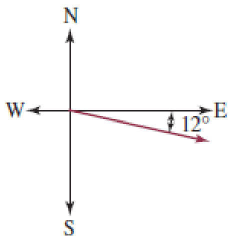
d)



e)



f)



4. Draw suitable diagrams to represent the following directions.

a) N45°E

b) S20°W

c) 028°T

d) 270°T

e) S60°E

f) 106°T

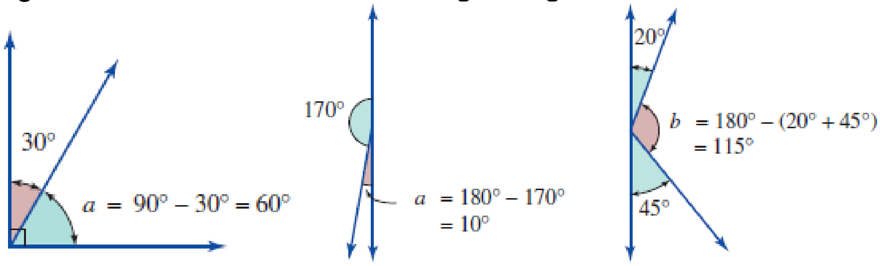
ANGLES

Angles are measured in degrees (°). In navigation, accuracy can be critical, so fractions of a degree are also used. For example, a cruise ship travelling 1000 kilometres on a course that is out by half a degree would miss its destination by almost 9 kilometres.

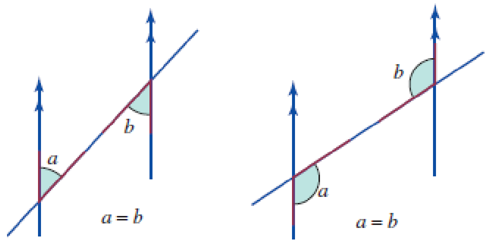
Some geometry (angle) laws

The following angle laws will be valuable when finding unknown values in the applications to come. Often we will need the laws to convert given directional bearings into an angle in a triangle.

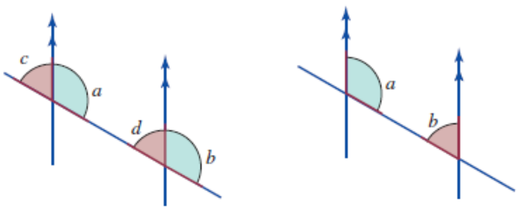
- Two or more angles are *complementary* if they add up to 90°.
- Two or more angles are *supplementary* if they add up to 180°.
- An angle of 180° is also called a *straight angle*.



For *alternate* angles to exist we need a minimum of one pair of parallel lines and one transverse line. Alternate angles are *equal*.

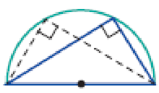


Other types of angles to be considered are *corresponding* angles, *co-interior* angles, *triangles in a semicircle* and *vertically opposite* angles.

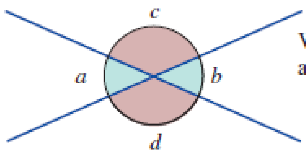


Corresponding angles are equal:
$a = b$
$c = d$

Co-interior angles are supplementary:
$a + b = 180°$



A triangle in a semicircle always gives a right-angled triangle.

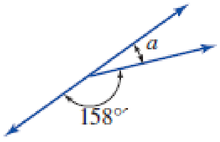


Vertically opposite angles are equal:
$a = b$
$c = d$

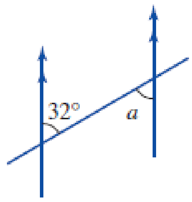
EXERCISE 6

1. Find the values of the pronumerals.

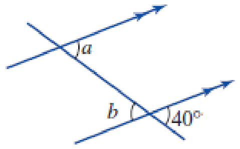
a)



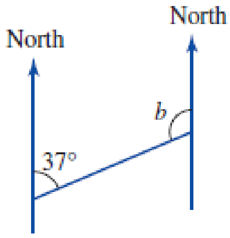
b)



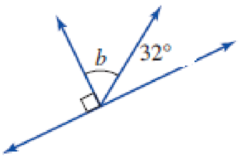
c)



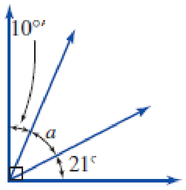
d)



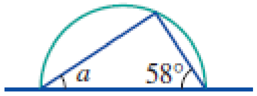
e)



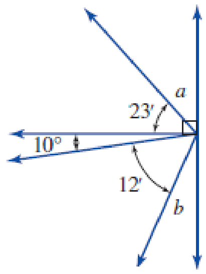
f)



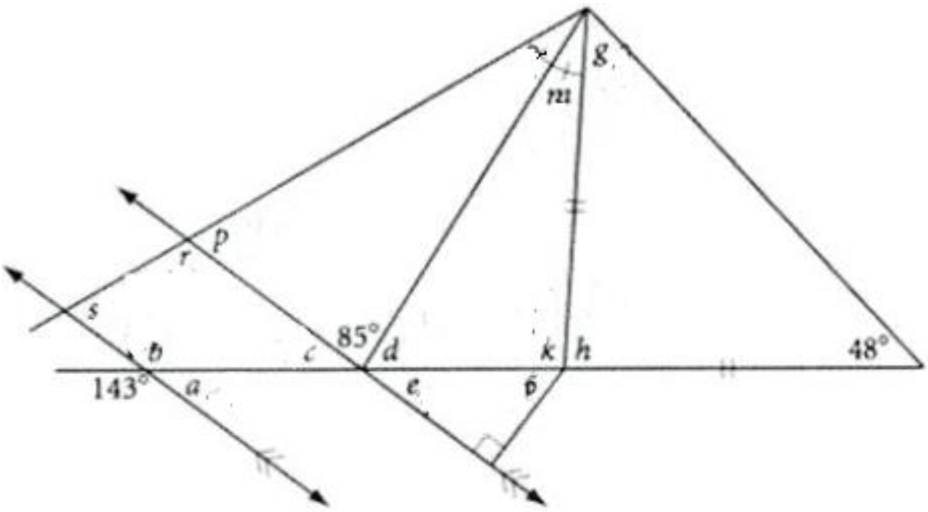
g)



h)



2. Find the value of the angles listed.



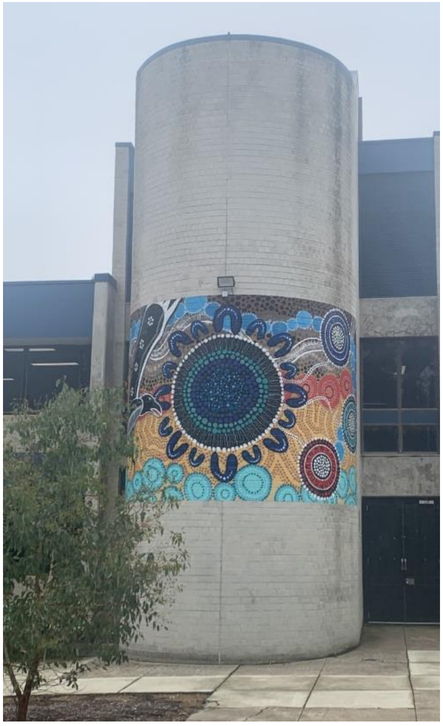
a = b = c = d = e = f =

g = h = m = p = r = s =

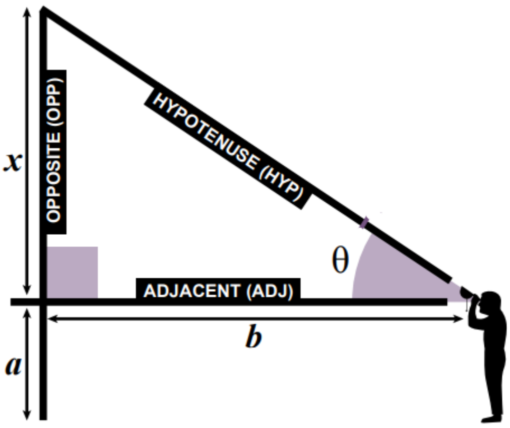
Working:

WEEK 5/6 INVESTIGATION

You will be using a clinometer and tape measure to calculate the height of the building shown. The building in the picture is at the front of the college.



You are required to calculate the height by taking 3 measurements. This can be achieved by either moving closer to the base of the building or further away from the base to vary your observation position. Your teacher will demonstrate the use of the clinometer to you.



Record your calculations in the table below:

	Observation 1	Observation 2	Observation 3
Distance from the base of the building			
Angle of elevation			
Height of the building (show working)			

What do you notice about the heights for each observation?

Suggest two reasons for this.

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation	Student completes the investigation of the brief to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and communications	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and techniques	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio task by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
		FINAL			**/20**

Student Reflection:
How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?