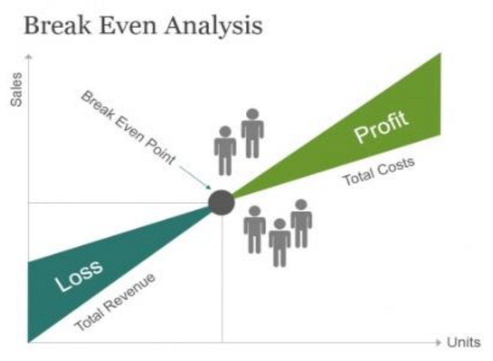
Week 13/14
Term 4
2023

HAWKER COLLEGE
Engage | Inspire | Achieve

MA2
Linear equations and their graphs

Goals



This fortnight we are going to:

- solve a pair of simultaneous linear equations, using technology when appropriate
- solve practical problems that involve finding the point of intersection of two straight-line graphs; for example, determining the break-even point where cost and revenue are represented by linear equations.

Theoretical Components

Resources:
PDF file: Week 13/14 Notes and Exercises
The clip covers break-even analysis
https://www.youtube.com/watch?v=Du07z79T-Js
Solving simultaneous equations by elimination
https://www.youtube.com/watch?v=8ockWpx2KKI

Knowledge Checklist
- Intersection of lines
- Break even analysis
- Solving simultaneous equations algebraically
  - Substitution method
  - Elimination method
- Interpreting simultaneous equations
- Graphing simultaneous equations
- Intersecting graphs (graphs that cross)
- Interpreting the intersection points
- Income v's costs

Order
1. Read through the notes and examples
2. Work through the exercises
3. Complete the investigation at the end of the booklet.
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Investigation

Complete the task at the end of the booklet and submit your work for checking. ☺

QFO

In **Week 15** you are to sit an **In-Class Task** worth 20% (with your weekly investigations) in your first lesson for the week (Monday). It is an **"open book"** task given under test conditions. You are allowed to bring in any of your notes and worked exercises and, of course, your calculator. You will need to come prepared with your Chromebook/laptop.

WEEK 13/14 NOTES & EXERCISES

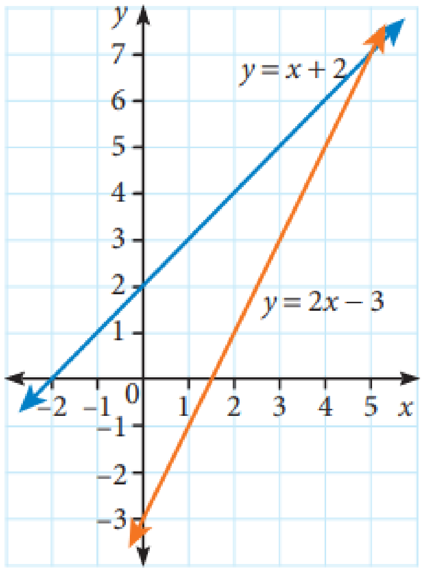
A pair of linear equations, such as $y = x + 2$ and $y = 2x - 5$, can be solved together to find the values of x and y that satisfy *both* equations. As they are solved at the same time, they are called **simultaneous linear equations**.

If simultaneous linear equations are graphed on the same Cartesian plane, their solution is the coordinates of the point where their graphs intersect.

The point where two or more graphs intersect is called the **point of intersection**.

You can check that your solution is correct by substituting the x and y values back into the original equations.

Sometimes the equations represent parallel lines that never intersect and therefore the simultaneous equations have no solution.



Point of intersection = (5, 7)

When using algebra to solve simultaneous equations, the aim is to obtain one equation with one unknown from two equations with two unknowns by various algebraic manipulations. This can be done in two ways – substitution and elimination – as outlined below.

SOLVING SIMULTANEOUS EQUATIONS - SUBSTITUTION METHOD

The method of substitution is easy to use when at least one of the equations represents one unknown in terms of the other.

To solve simultaneous equations using the method of substitution:

1. Check that one of the equations is transposed so that one of the unknowns is expressed in terms of the other.

2. Substitute the transposed equation into the second equation.

3. Solve for the unknown variable.

Example

Use the method of substitution to solve the following pair of simultaneous equations:

$y = 2x + 3$ and

$4x - y = 5$

1	Write down the equations, one under the other, and number them.	$y = 2x + 3$ [1] $4x - y = 5$ [2]
2	Substitute the expression $(2x + 3)$ from equation [1] for y into equation [2]. *Note*: By substituting one equation into the other, we are left with one equation and one unknown variable.	Substituting $(2x + 3)$ into [2]: $4x - (2x + 3) = 5$
3	Solve for x.	
	(a) Expand the brackets on the LHS of the equation.	$4x - 2x - 3 = 5$
	(b) Simplify the LHS of the equation by collecting like terms.	$2x - 3 = 5$
	(c) Add 3 to both sides of the equation.	$2x - 3 + 3 = 5 + 3$ $2x = 8$
	(d) Divide both sides of the equation by 2.	$\frac{2x}{2} = \frac{8}{2}$ $x = 4$
4	Substitute 4 in place of x into [1] to find the value of y.	Substituting $x = 4$ into [1]: $y = 2 \times 4 + 3$
5	Evaluate.	$= 8 + 3$ $= 11$
6	Answer the question.	Solution: $x = 4$, $y = 11$ or solution set $(4, 11)$.

ELIMINATION METHOD

As the name suggests, the idea of the elimination method is to eliminate one of the variables. This is done in the following way.

1. Choose the variable you want to eliminate.

2. Make the coefficients of that variable equal in both equations.

3. Eliminate the variable by either adding or subtracting the two equations.

Once this is done, the resulting equation will contain only one unknown which then can be easily found.

Example

Use the elimination method to solve the following:

$2x + 3y = 4$ and

$x - 3y = 2$

Solution

1. Write down the equations, one under the other, and number them.

   $2x + 3y = 4$ [1]

   $x - 3y = 2$ [2]

2. Add equations [1] and [2] in order to eliminate y.

   Note: y was eliminated since the coefficients of y in both equations were equal in magnitude and opposite in sign.

   [1] + [2]:

   $2x + 3y = 4$

   $+ (x - 3y = 2)$

   $3x = 6$

3. Divide both sides of the equation by 3.

   $\frac{3x}{3} = \frac{6}{3}$

   $x = 2$

4. Substitute the value of x into equation [2].

   Note: $x = 2$ may be substituted in either equation.

   Substituting $x = 2$ into [2]:

   $x - 3y = 2$

   $2 - 3y = 2$

5. Solve for y.

   (a) Subtract 2 from both sides of the equation.

   $2 - 2 - 3y = 2 - 2$

   $-3y = 0$

   (b) Divide both sides of the equation by -3.

   $\frac{-3y}{-3} = \frac{0}{-3}$

   $y = 0$

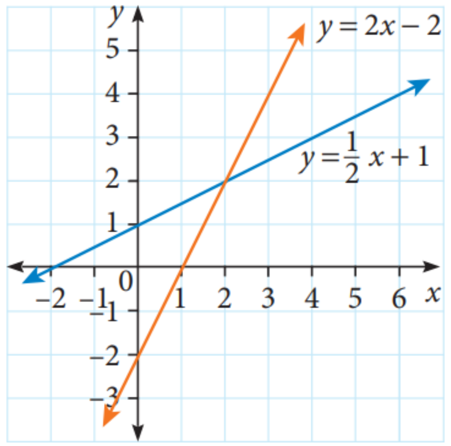
6. Answer the question.

   Solution: $x = 2$, $y = 0$ or solution set $(2, 0)$.

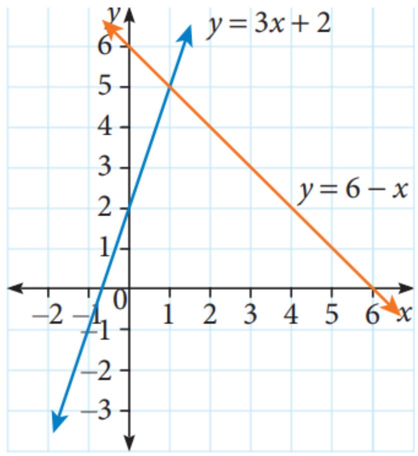
EXERCISE 1

1. Find the point of intersection of each pair of simultaneous equations given their graphs below.

a)



b)



2.

The graph shows the intersection of two linear functions.

a The equations of the lines are:

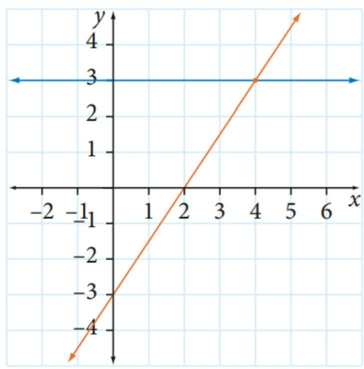
A $y = \frac{2}{3}x - 3$ and $x = 3$

B $y = \frac{3}{2}x - 3$ and $y = 3$

C $y = -\frac{2}{3}x - 3$ and $y = 3$

D $y = \frac{3}{2}x - 3$ and $x = 3$

E $y = \frac{2}{3}x - 3$ and $y = 3$



b The coordinates of their point of intersection are:

A (0, 3) **B** (2, –3) **C** (4, 3) **D** (2, 0) **E** (3, 4)

3. Solve the following simultaneous equations by the process of **substitution**.

a) $y = 2x + 3$

$6x + y = 11$

b) $y = 3x - 6$

$y = 16 + 5x$

c) $2y - 6 = x$

$7x + 3y = -25$

d) $x = 5 - 4y$

$2y - 3x = 13$

4. Solve the following simultaneous equations by **elimination**.

a) $2x + y = 3$

$4x - y = -9$

b) $x + 2y = 5$

$x - 4y = 2$

c) $3x - 2y = -1$

$3x - 6y = -9$

d) $2x - y = 0$

$2x - 4y = -9$

SOLVING PROBLEMS USING SIMULTANEOUS EQUATIONS

Simultaneous equations are used to solve a variety of problems containing more than one unknown. Here is a simple algorithm which can be applied to any of them:

1. Identify the variables.

2. Set up simultaneous equations by transforming written information into algebraic sentences.

3. Solve the equations by using the substitution, elimination or graphical methods.

4. Interpret your answer by referring back to the original problem.

Example

Two hamburgers and a packet of chips cost $8.20, while 1 hamburger and 2 packets of chips cost $5.90. Find the cost of a packet of chips and a burger.

(1) Define the two variables.	Let x = the cost of one hamburger. Let y = the cost of a packet of chips.	
(2) Formulate an equation from the first sentence and call it [1]. *Note*: 1 hamburger costs $x, 2 hamburgers cost $2x. Thus, the total cost of cost of 2 hamburgers and 1 packet of chips is $2x + y$ and it is equal to $8.20.	$2x + y = 8.20$	[1]
(3) Formulate an equation from the second sentence and call it [2]. *Note*: 1 packet of chips costs $y, 2 packets cost $2y. Thus, the total cost of 2 packets of chips and 1 hamburger is $x + 2y$ and it is equal to $5.90.	$x + 2y = 5.90$	[2]

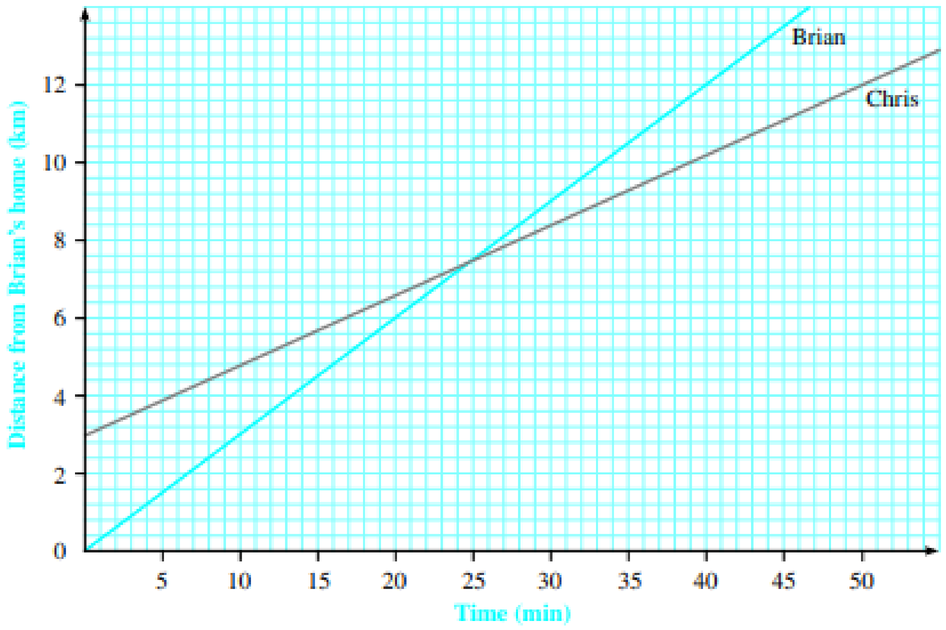
Solve these by the elimination or substitution method.

$x = 3.5$, $y = 1.2$

Answer the question and include appropriate units.	A hamburger costs $3.50 and a packet of chips costs $1.20.

EXERCISE 2

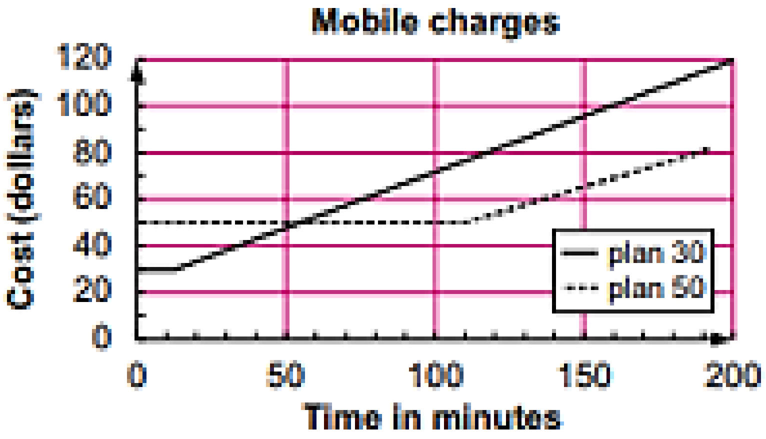
1. Chris jogged to the park while Brian rode his bike. Chris was 3 km ahead of Brian and ran at a speed of 10.8 km/h, while Brian rode at a speed of 18 km/h. The travel graph shows their journeys, with the distance of both from Brian's home at different times.



a) Did Brian catch up to Chris? If so when, and where?

b) If the park was 12 km away from Brian's home, who got there first and by how much time?

2. Rebecca is deciding which mobile phone plan to use. She is comparing the Plan 30 and Plan 50 rates. Plan 30 has minimum $30 per month with $5 of free calls. Calls cost 24 cents per 30 seconds. Plan 50 has minimum $50 per month with $40 of free calls. Calls cost 19 cents per 30 seconds. The graph of these plans is shown below.



a) After how many minutes of calls are the costs of each program equal?

b) Lorena averages 35 minutes of calls per month. Which plan should she use? Why?

c) Jenny uses her phone to receive calls and makes at most three 1 minute calls per month. Which plan should she use? Explain.

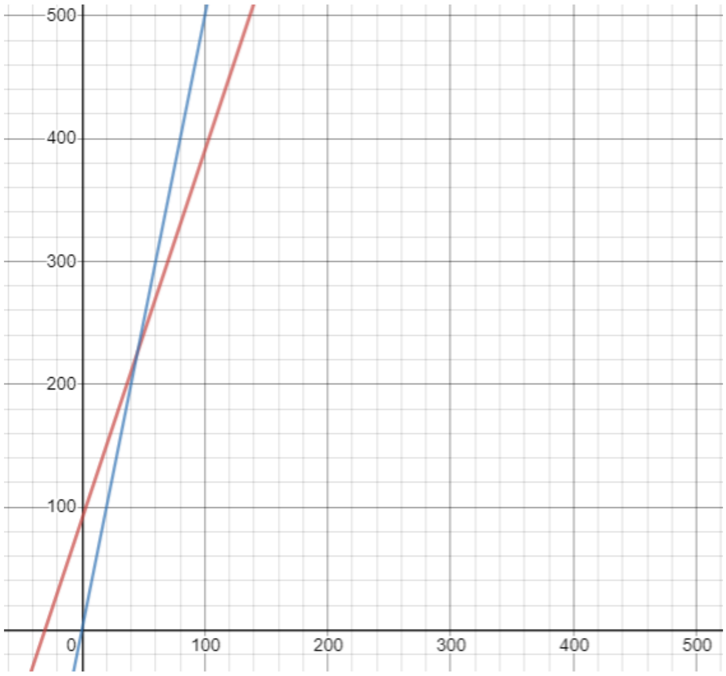
3. At the end of the day, two shop assistants compare their sales. One sold 5 toasters and 2 sandwich-makers for a total of $149.65, while the other sold 3 of each for a total value of $134.70. Find the price of each item.

4. Spiro empties his piggy bank. He has 42 coins, some of which are 5c coins and some of which are 10c coins, to the total value of $2.50. How many 5c coins and how many 10c coins does he have?

5. A rectangle's length is 2 cm more than its width. If the perimeter of a rectangle is 24 cm, find its dimensions and, hence, its area.

6. The cost of manufacturing yo-yos is given by the formula C = 3n + 90, where n is the number of yo-yos and C is cost in dollars. The revenue from selling yo-yos is given by the formula R = 5n, where n is the number of yo-yos sold and R is the sales in dollars.

a) Graph both formulas on the same axes for values of n from 0 to 50.
(Put labels on the graph already drawn for you)



b) What is the cost of manufacturing 20 yo-yos?

c) What is the revenue from selling 20 yo-yos?

d) How much does one yo-yo sell for?

e) For what value of n does the cost equal the revenue?

7. Megan and Jane both sell kitchenware and earn commissions. Megan's commission is $240 plus 5% of her sales, while Jane's commission is 20% of her sales. These rates can be expressed by the formulas:

Megan's commission $C = 0.05x + 240$
Jane's commission $C = 0.2x$

where x represents the value of sales in dollars.

a) Graph both commission formulas on the same axes for values of x from $0 to $2400.

b) If each salesperson sells $1200 worth of kitchenware, who earns more?

c) For what value of sales does Megan earn more than Jane?

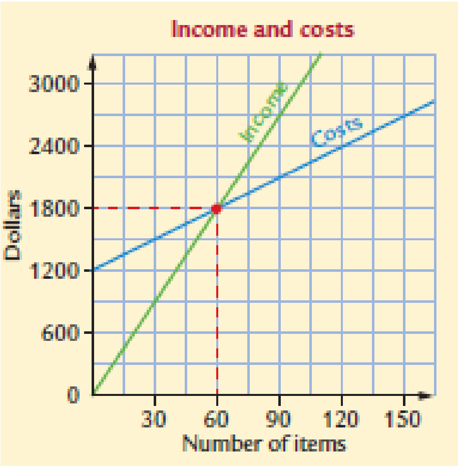
d) For what value of sales does Megan earn the same as Jane?

BREAK-EVEN ANALYSIS

Break-even is the point where a business's costs are the same as the money it receives from sales. Knowing the break-even point is essential to making a profit. If a business is not making a profit, it won't last long!

Example

A small business's total fixed costs are $1200 per week and its variable costs are $10 per item it produces. Each item produced is sold for $30. How many items does the business need to sell each week to break even?



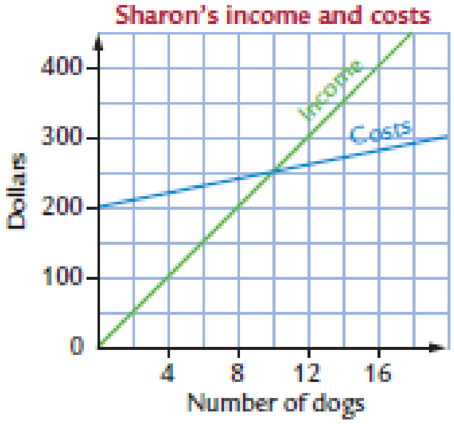
Solution

A graph is the easiest way to solve this problem. The line **Costs** shows the business's cost of producing different numbers of items. The line **Income** shows the income the business receives from selling different numbers of items. The point where the lines cross is the business's break-even point.

When the business sells 60 items per week, the income and costs are equal. If the business sells more than 60 items, it will make a profit. If it sells less than 60 items, it will make a loss.

EXERCISE 3

1. Sharon has a dog-washing business. The graph shows her weekly expenses and income.



a) Explain how you know that Sharon's fixed costs are $200 per week.

b) How much does Sharon charge for washing dogs?

c) How many dogs does Sharon have to wash each week to break even?

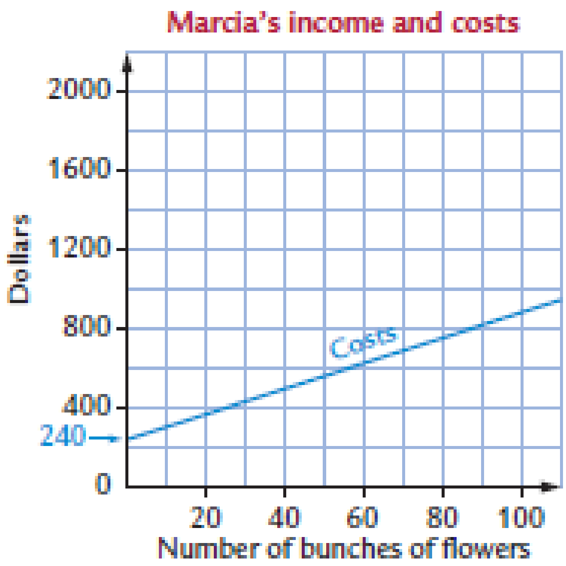
d) How much profit does Sharon make in a week if she washes 16 dogs?

2. Marcia sells bunches of flowers from her street flower stall. She sells each bunch for $15.

a) Complete this table of values to show the money she will receive from selling different numbers of bunches of flowers.

Bunches of flowers sold	0	15	40	50	65	90
Money received						

b) The graph shows Marcia's weekly costs for selling flowers. Use the graph to show Marcia's income line on it.

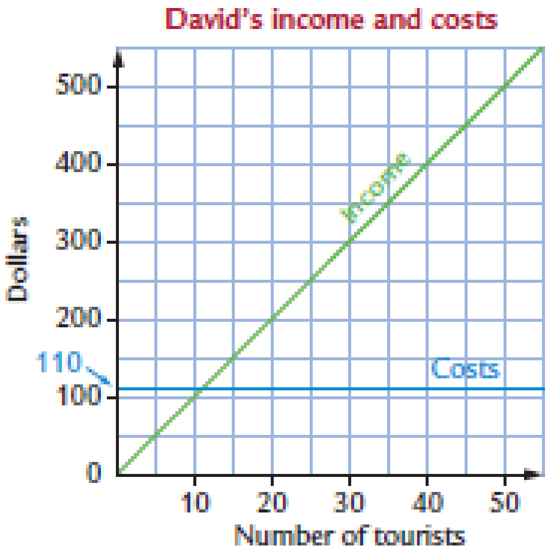


c) How many bunches of flowers does Marcia need to sell each week to break even?

d) How much are Marcia's fixed costs per week?

e) Calculate Marcia's profit when she sells 80 bunches of flowers.

3. During the summer tourist season, David shows tourists the sights of his area in a horse-drawn carriage. This graph shows his weekly costs and income. Use the graph to answer the following questions.



a) How much does David charge each tourist he takes in his carriage?

b) How many tourists does he need to drive each week to break even?

c) Suggest a reason why David's cost line is horizontal.

WEEK 13/14 INVESTIGATION

Jon's coffee shop will be open for 7 days per week and during Friday night shopping. From his market research, Jon determined that his monthly rent will be $7400 and his other monthly fixed costs will total $2100. He plans to work in the shop himself and to have four employees working a total of 120 hours per month (altogether, not each) at $15 per hour. Jon's superannuation payment for labour is $162 per month.

Jon thinks that it will cost 46 cents to make a cup of coffee and 52 cents (each item) to make cakes and muffins. He plans to have an opening price of $5.20 for a coffee with cake or muffin.

Jon's target number of sales per month is 3650.

- Calculate the number of sales required to **break even**: use total cost of monthly expenses divided by (price of typical coffee and cake subtract cost of ingredients)
- Calculate Gross profit after GST: use total income minus total costs
  - Total income is (target number of sales × price of coffee and cake × 10/11)
  - Total costs are [total cost of monthly expenses plus (target number of sales × ingredient costs)]   Note: ×10/11 is the way of accounting for GST.

Jon's Coffee Shop

Jon's monthly expenses		
Rent		
Other fixed costs		
Monthly Labour		
Hours of employee labour		
Average cost of labour per hour		
Superannuation payment for labour		*
Total cost of monthly expenses and labour		*
Average ingredient costs		
Coffee		
Cakes and muffins		
Income		
Price of a typical coffee and cake or muffin		
Break even		
Number of sales required monthly to break even		*
Target number of sales per month		
Gross profit after GST		*

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation	Student completes the investigation of the brief to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and communications	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and techniques	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio task by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
		FINAL			**/20**

Student Reflection:
How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?