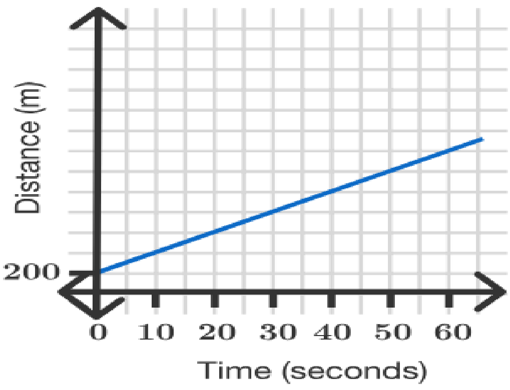
Week 12
Term 4
2023

HAWKER COLLEGE
Engage | Inspire | Achieve

MA2
Linear equations and their graphs

Goals



This week we are going to:

- construct straight-line graphs both with and without the aid of technology
- determine the slope and intercepts of a straight-line graph from both its equation and its plot
- interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation
- construct and analyse a straight-line graph to model a given linear relationship; for example, modelling the cost of filling a fuel tank of a car against the number of litres of petrol required.

Theoretical Components

Resources:
PDF file: Week 12 Notes and Exercises
This clip shows how to sketch a linear graph using a table of values.
https://www.youtube.com/watch?v=7sg8h0Y8oZk
This link shows how to sketch a graph using the form $y = mx + b$
https://www.youtube.com/watch?v=NAblGVxxJZo
A good overview
https://www.khanacademy.org/math/algebra-home/alg-linear-eq-func/modal/v/2-variable-linear-equations-graphs

Knowledge Checklist
- Concept of gradient (including horizontal and vertical lines)
- Calculating gradient using rise over run
- Positive and negative gradient
- Table of values to draw a linear graph
- Using the y-intercept and gradient to sketch a linear function
- Linear relationship between two variables

Order
1. Read through the notes and examples
2. Work through the exercises
3. Complete the investigation at the end of the booklet.
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Investigation

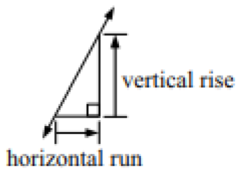
Complete the task at the end of the booklet and submit your work for checking. ☺

QFO

LINEAR GRAPHS – GRADIENT AND INTERCEPT

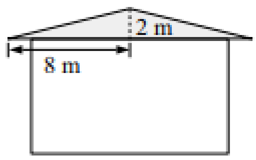
The words slope and gradient are used when talking about the steepness of a line. Gradient or slope is associated with house roofs, escalators, hills, etc. The steeper the slope the larger the gradient. In order to compare the slopes in different situations there is a standard approach.

i.e., $\textbf{slope} = \dfrac{\textbf{vertical rise}}{\textbf{horizontal run}}$



The following illustrations indicate slopes of varying amounts.

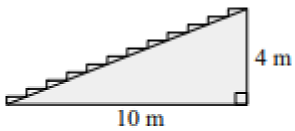
House Roof



$\text{slope} = \frac{2}{8}$

$= \frac{1}{4}$

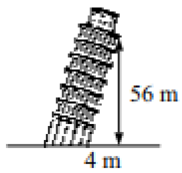
Escalator



$\text{slope} = \frac{4}{10}$

$= \frac{2}{5}$

Leaning Tower of Pisa



$\text{slope} = \frac{56}{4}$

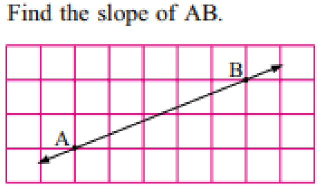
$= 14$

Note: For a horizontal line the vertical rise is 0, therefore the slope is zero.

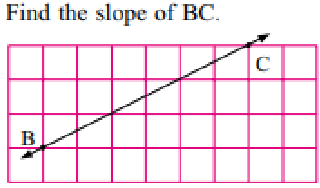
When line segments are drawn on graph paper we can easily determine the slope of the line segment by drawing horizontal and vertical lines to complete a right-angled triangle.

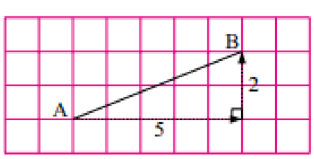
Examples

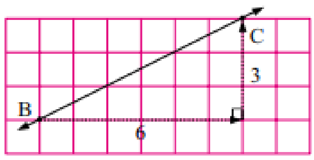
a Find the slope of AB.



b Find the slope of BC.







a slope of AB $= \dfrac{\text{vertical rise}}{\text{horizontal run}}$

$= \frac{2}{5}$

b slope of BC $= \dfrac{\text{vertical rise}}{\text{horizontal run}}$

$= \frac{3}{6}$

$= \frac{1}{2}$

EXERCISE 1

1. Complete the sentences.

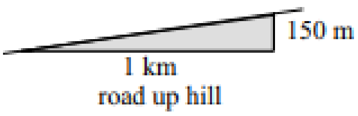
a) The slope of a horizontal line is ______________________

b) The slope of a vertical line is _______________________

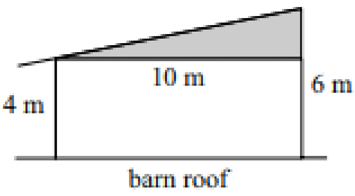
c) As the line segments becomes steeper their slopes ___________

2. Find the slope of the following.

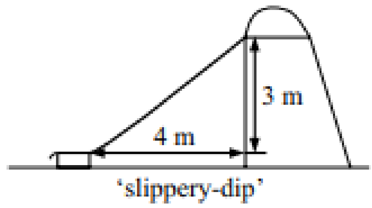
a)



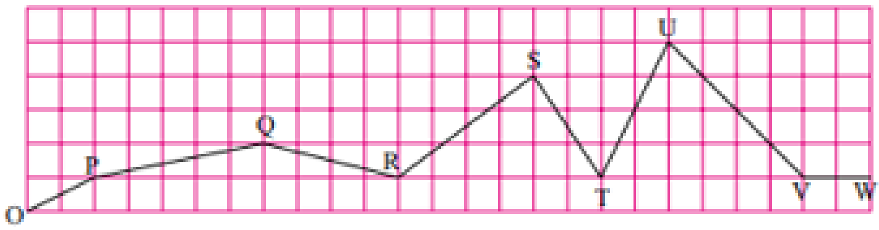
b)



c)



3. The following diagram represents a path crossing through the countryside.



a) Indicate the uphill parts of the path.

b) Indicate the downhill parts of the path.

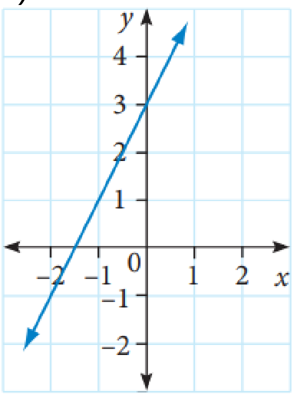
c) Where is the steepest positive slope?

d) Where is the steepest negative slope?

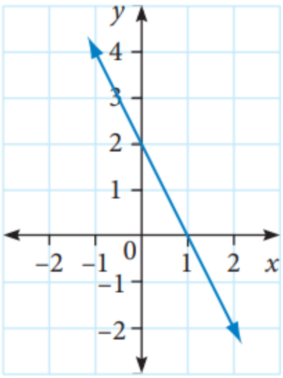
e) Where is the slope equal to 0?

4. Find the gradient of these lines and the y-intercept (where they cross the vertical axis).

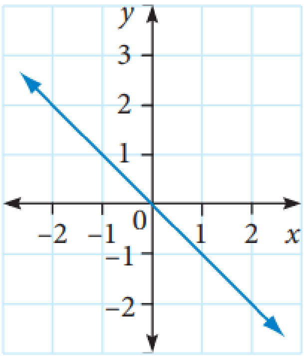
a)



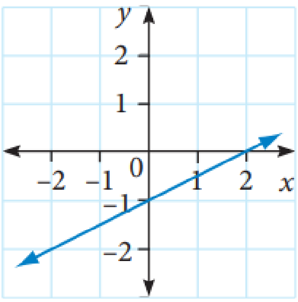
b)



c)



d)



LINEAR GRAPHS

The rules or equations for straight-line graphs may all be written in a similar form. The convention for writing the rule for straight line graphs is;

$$y = mx + b$$

This relationship connects every point on the straight-line graph.

x	is the independent variable as any value may be used.
y	is the dependent variable as it depends on the value of x.
m	is the gradient.
b	is the intercept with the vertical axis, the y intercept in this case.

Example

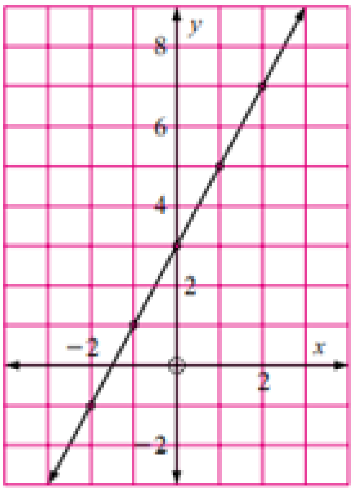
Draw the graph modelled by the equation a) $y = 2x + 3$ b) $y = -3x + 5$

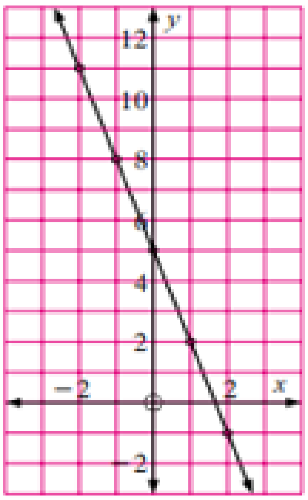
Use a table of values

x	-2	-1	0	1	2
y	-1	1	3	5	7

x	-2	-1	0	1	2
y	11	8	5	2	-1

Plot the points and draw a line through them





The equation of a straight line can (usually) be written as *y* = mx + b where m is the gradient and b is the y-intercept. Therefore, any equation involving x and y which can be rearranged into that form describes a straight line.

EXERCISE 2

1. Write the equation of the line for each of the graphs from page 4. $\boldsymbol{y = mx + b}$

a) b)

c) d)

2. Which of the following describe or determine a straight line? You need to justify your answer in each case.

In some of these drawings a graph may be the best solution. You could use graphing software such as *Desmos.*

The equation of a straight line can (usually) be written as y = mx + b where m is the gradient and b is the y-intercept. Therefore, any equation involving x and y which can be rearranged into that form describes a straight line.

Equation	**Is it a straight line?**	**Explanation/give sketch**
a) $4x - 2y = 6$		
b) $y = 2$		
c) $y = \frac{3}{2}x$		

d) $y = 3 - 2x$		
e) $y = x^2 + 2$		
f) $x = -2$		
g) the point (0, -1) and the constant gradient 3		
h) $x = 7y + 5$		
i) $y - 8 = 3(x - 3)$		

j) $y^2 = x^2$		
k) $\frac{1}{3}y - x + \frac{1}{3} = 0$		
l) $xy = 1$		
m) $y^2 - 4xy + 4x^2 = 0$		
n) the points ($\frac{1}{2}$, 2), (1, 1) and ($\frac{3}{2}$, 0)		

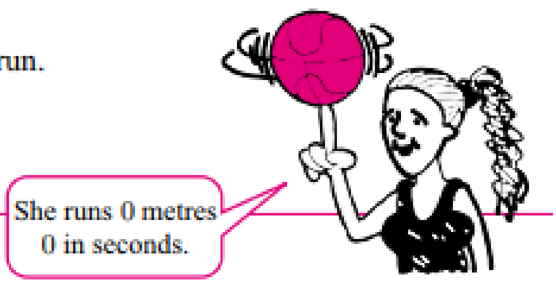
DIRECT VARIATION

The gradient and intercept of linear graphs have meaning in practical situations. A relationship that gives a straight line when graphed is a direct relationship between the two variables.

Example

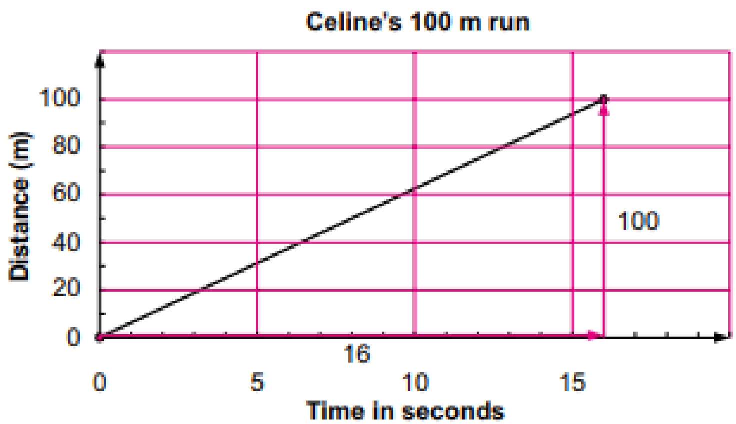
At the athletics carnival Celine runs the 100m in 16 seconds. It is assumed that she runs at a constant rate.

- **a** Sketch a straight-line graph to model Celine's run.
- **b** Find the gradient.
- **c** Explain the meaning of the gradient.



a Use two points to draw the graph.
They are (16, 100) and (0, 0).

This is the graph.



b Draw in a triangle.

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{100}{16} = 6.25$$

The units for the gradient in this case are m/s.

c The gradient is Celine's speed in metres per second. She runs at an average speed of 6.25 m/s.

EXERCISE 3

1. Christopher runs 100 metres in 20 seconds.

a) Draw a graphical model of Christopher's run.

b) How long would it take to run 30 metres?

c) Find the gradient and hence Christopher's speed in metres per second.

2. Craig drives 200 km in 4 hours.

a) Draw a graphical model of Craig's drive.

b) How far does Craig drive in $1\frac{1}{2}$ hours?

c) Find the gradient.

d) What are the units of the gradient in this case?

e) Expand the graph to 1000 km. This is called extrapolation. Is the graph still accurate as a model for Craig's drive? Explain.

LINEAR MODELLING

Example

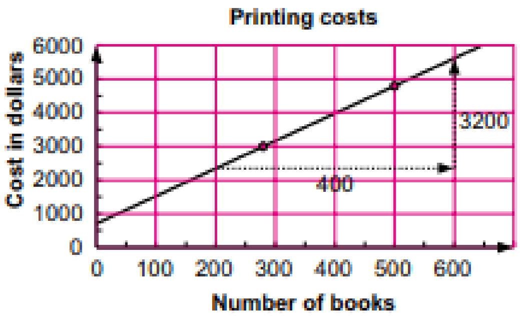
Use this graph of printing costs to find

a the cost to print 500 books

b the number of books that can be printed for $3000.

c Find the gradient. What is its meaning?

d Find the y intercept. What is its meaning?



Solutions

a 500 books would cost about $4800.

b $3000 would buy about 280 books

c Draw in a triangle to find the gradient.

$$\text{gradient} = \frac{3200}{400} = 8$$

The gradient is 8. The units are dollars per book.
This means that the gradient is the cost per book.

d The y-intercept is approx 800.This means the cost to print 0 books is $800. This is the set up cost.

EXERCISE 4

1. Ruth exhibits her art work at Gino's Art Gallery. On all works sold through the gallery, Gino charges a commission calculated using a linear formula. In the past Gino received $75 commission on a $1000 painting and $125 on a $3000 sculpture. On this basis, Ruth expects to be charged $200 on her most recent sale, a $4000 sculpture. What commission does Gino actually receive on this sale?

2. Here is a graph modelling taxi charges.

a) Find the cost of travelling 25 km.

b) How far can you travel for $15?

c) Find the gradient (m). What is the meaning of the gradient?

d) Find the intercept on the vertical axis (b). What is its meaning?

e) Write an equation for the model in the form of $\boldsymbol{C = b + md}$
where C is the cost in dollars ($) and d is the distance in kilometres (km).

3. Here is a graph modelling catering charges.

a) How much would it cost for 35 people?

b) How many people could eat for $300?

c) Find the gradient. What is the meaning of the gradient?

d) Find the intercept on the vertical axis. What is its meaning?

e) Write an equation for the model.

WEEK 12 INVESTIGATION

Here are the equations of 12 straight lines.

$y = 4x + 4$	$4y = x + 3$	$y = 8x - 3$	$y + 4x + 6 = 0$
$3y = 2x - 8$	$y + 6x = 11$	$y + 8x = 6$	$2y + 8 = 3x$
$2y + x = 4$	$2y = 8x + 3$	$y = 6x - 4$	$y + x + 8 = 0$

1. Rewrite each equation in the form $y = mx + b$

2. These 12 straight lines can be divided up into six pairs, each pair matching one of the following descriptions. Sort them into the correct pairs and complete the final description.

- These lines are parallel.
- These lines are perpendicular.
- These lines have the same y-intercept.
- These lines have the same x-intercept.
- These lines both go through the point (1,5).
- These lines …

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation	Student completes the investigation of the brief to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and communications	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and techniques	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio task by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
		FINAL			**/20**

Student Reflection:
How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?