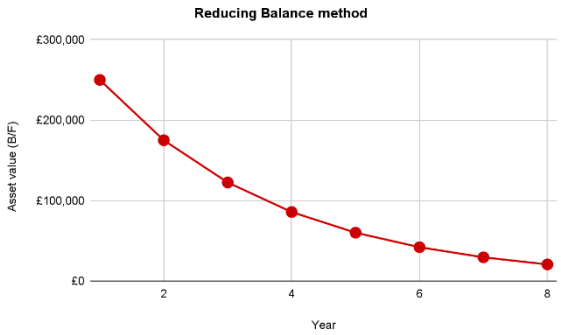
Week 11
Term 1
2024

HAWKER COLLEGE
Engage | Inspire | Achieve

MA3
Growth and decay in sequences

Goals



This week we are covering:

- compound interest (appreciation)
- reducing balance depreciation

Theoretical Components

Resources:
PDF file: Week 11 Notes & Exercises

Formulas:

Compound interest

$$A = PR^n$$

where

R = 1 + $\frac{r}{100}$
A = amount in the account, $
P = principal, $
r = interest rate (%) per compounding period
n = the number of compounding periods

Reducing balance depreciation

$$A = PR^n$$

where

R = 1 - $\frac{r}{100}$
A = value, $
P = principal (cost), $
r = depreciation rate per period
n = the number of depreciating periods

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Knowledge Checklist

- Compound interest
- Depreciation

Investigation

Complete the task at the end of the booklet and submit your work for checking. ☺

Quiz/forum/ other

Remember to check hawkermaths.com **for each week's learning brief.**
Make sure you have joined Google Classroom. If you have not, see your teacher.

MATHEMATICAL APPLICATIONS 3

WEEK 11 NOTES & EXERCISES

COMPOUND INTEREST

Consider the case where a bank pays compound interest of 5% per annum on an amount of $20000. The amount is invested for 4 years and interest is calculated yearly. Compound interest receives its name because the interest which is earned is paid back into the account so that the next time interest is calculated, it is calculated on an increased amount. There is a compounding effect on the money in the account. If we calculated the amount in the account mentioned above each year, we would have the following amounts.

Start $20000		
After 1 year	$20 000 × 1.05	= $21 000
After 2 years	$20 000 × 1.05 × 1.05	= $22 050
After 3 years	$20 000 × 1.05 × 1.05 × 1.05	= $23 152.50
After 4 years	$20 000 × 1.05 × 1.05 × 1.05 × 1.05	= $24 310.13

The amounts 20000, 21000, 22050, 23152.50, 24310.13, ... form a geometric sequence where a = 20000 and r = 1.05.

We need to be a little careful, however, in using the formula $\boldsymbol{t_n = ar^{n-1}}$ in calculating compound interest. This is because the original amount in the account, that is, $20000, in terms of the geometric sequence would be referred to as t_1 or *a*. In banking terms, t_1 would represent the amount in the account after the first lot of interest has been calculated and added in. To be clear and to be safe, it is best to use the following formula for compound interest.

$$\boldsymbol{A = PR^n}$$

where $R = 1 + \frac{r}{100}$

A = amount in the account, $
P = principal, $
r = interest rate per period (that is, per year or quarter etc.), %
n = the number of periods during the investment.

Example

Helen inherits $60000 and invests it for 3 years in an account which pays compound interest of 8% per annum compounding each 6 months.

a) What will be the amount in Helen's account at the end of 3 years?
b) How much will Helen receive in interest over the 3-year period?

Solution

1 This is an example of compound interest.

Use $A = PR^n$, where $R = 1 + \frac{r}{100}$. Interest is calculated each 6 months so, over 3 years, there are 6 periods: $n = 6$. Interest is 8% per year or 4% per 6 months.
So, $r = 4\%$.

2 Write your answer.

a $P = 60\,000$
$n = 6$ half years
$r = 4\%$ per half year

So, $R = 1 + \frac{4}{100}$
$= 1.04$
$A = PR^n$
$= 60\,000(1.04)^6$
$= 75\,919.14$

At the end of 3 years, Helen will have a total amount of $75 919.14.

1 Interest equals the amount in the account at the end of 3 years, less the amount in the account at the start of the investment.

2 Write your answer.

b Interest = Total amount − Principal
$= \$75\,919.14 - \$60\,000$
$= \$15\,919.14$

Amount of interest earned over 3 years is $15 919.14.

Example

Jim invests $16000 in a bank account which earns compound interest at the rate of 12% per annum compounding every quarter. At the end of the investment, there is $25616.52 in the account. For how many years did Jim have his money invested?

Solution

1 We know the value of A, P, r and R.
We need to find n using the compound interest formula.

$A = 25\,616.52$
$P = 16\,000$
$r = \frac{12}{4}$
$= 3\%$ per quarter
and so $R = 1 + \frac{3}{100}$
$= 1.03$
Now, $A = PR^n$
So, $25\,616.52 = 16\,000(1.03)^n$
$1.601 = 1.03^n$

2 Try some different values of n.

Let $n = 5$ $1.03^5 = 1.159$
Let $n = 10$ $1.03^{10} = 1.344$
Let $n = 15$ $1.03^{15} = 1.558$
Let $n = 16$ $1.03^{16} = 1.605$

3 Write your answer.

It will take 16 periods where a period is 3 months. So, it will take 48 months or 4 years.

EXERCISE 1

1. $13,000 is invested in an account which earns compound interest of 8% p.a, compounding quarterly.

a) After 5 years, how much is in the account?

b) How much interest was earned in that period?

2. $10,000 is invested in an account which earns compound interest of 10% per annum. Find the amount in the account after 5 years if the interest is compounded monthly.

3. $10,000 is invested in an account which earns compound interest of 10% per annum. Find the amount in the account after 5 years if the interest is compounded daily. Compare your answer to that in the previous question.

4. In an account earning compound interest of 8% per annum compounding quarterly, an amount of $6,000 is invested. When the account is closed, there is $7,609.45 in the account. For how many years was the account open?

5. Helena receives $15,627.12 after closing an investment account which earned compound interest of 9% per annum compounding every 6 months. If Helena originally deposited $12,000 in the account, for how long was it in the account?

DEPRECIATION

We have looked at compound interest, a situation where amounts APPRECIATE, or increase in value. However, items may also decrease in value- this is known as DEPRECIATION. Cars (usually), machinery, computers and electronics are examples of things that depreciate. In other words, they're worth less over time than when you bought them.

Straight Line Depreciation

Straight line depreciation is similar to simple interest in that the depreciation (amount lost) is the same each year.

Reducing Balance Depreciation

The more common form of depreciation is reducing-balance depreciation. The rules for calculating this kind of depreciation are similar to calculating compound interest. The formula is just slightly different.

The depreciation formula is:

$$\boldsymbol{A = PR^n} \text{ where } R = 1 - \frac{r}{100}$$

Example

A microwave that costs $700 depreciates at 20% pa.

$R = 1 - \frac{20}{100} = 0.8$

Thus $A = 700 \times 0.8^n$

a) What is it's value after 3 years?

$A = 700 \times 0.8^3 = \$358.40$

b) How long will it take for the microwave to be worthless? Let's assume 'worthless' means less than $1.

$1 = 700 \times 0.8^n$

By trial and error n = 30 (value $0.80)

EXERCISE 2

1. James purchased a $18,900 motorbike, which depreciates at a compounded rate of 15% p.a.

a) What is the amount of the depreciation for the first year?

b) What is the expected value after the first year?

c) What is the depreciation for the second year?

d) What is the expected value after the second year?

e) What is the total depreciation over the two years?

f) What is the percentage of the original value remaining after two years?

2. Sally purchased an iPod for $800, which depreciates at 15% p.a. What is its resale value after 4 years?

3. The government wants to decrease its spending on job creation. Currently it is spending $160 million and will decrease it by 6.15% p.a. over the next 10 years. Calculate the government's spending in 10 years' time. Round your answer to the nearest dollar.

4. A netbook depreciated by 28% p.a and was valued at $900 after 7 years. What was the original price?

INVESTIGATION WEEK 11

1. Carla invests $20000 for three years at 5.7% per annum compounded annually. At the end of this time she reinvests the amount returned from her three year investment plus an additional $4000 she has saved, at 5.8% per annum compounded annually for a further two years. How much does she receive from her investment at the end of the five years?

2. Consider the compound interest formula $\boldsymbol{A = PR^n}$ and investigate the effect changing the value of P has on the future value, A.
Illustrate your answer with an example.

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation	Student completes the investigation of the brief to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and communications	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and techniques	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and investigation by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
		FINAL			**/20**

Student Reflection:
How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?