Week 11/12
Term 4
2023

HAWKER COLLEGE
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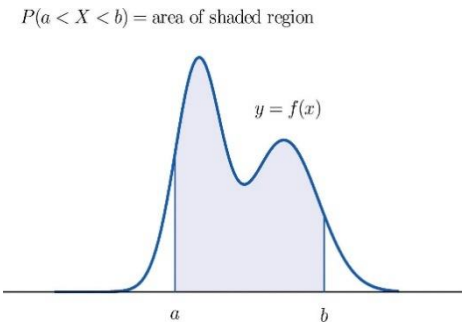
Learning Brief

SMM4

Goals



This fortnight we are going to:

- Understand probability distribution for continuous random variables
- Recognise situations when the normal distribution applies, learn how to solve problems involving the normal distribution, and when to use the normal distribution to approximate the Binomial or Poisson distribution
- Recognise and use the formula to compute probabilities
- Use the CAS to compute probabilities
- Understand the assumptions on which the normal model is based
- Understand the probability limits of *almost certainly* and *very probably*, that is, the three and two sigma limits

Theoretical Components

Make notes on the following chapters:

Maths Quest 12 Mathematical Methods

- 12E - The normal distribution
- 12F - The standard normal distribution
- 12G - The inverse cumulative normal distribution

Reasoning and Data

- 6.1 The normal distribution
- 6.2 Standard normal curve
- 6.3 Normal approximation to binomial distribution
- 6.4 Probability limits for a single value of the normal variable
- 6.5 Probability limits for the sample mean of n values of the variable

Normal Curve:

- https://www.youtube.com/watch?v=McSFVzc8Swk

Normal Distribution:

- https://www.intmath.com/counting-probability/14-normal-probability-distribution.php
- https://stattrek.com/probability-distributions/normal?Tutorial=AP

1. $f(x) \geq 0$ for all x
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$
3. $P(a \leq x \leq b) = \int_{a}^{b} f(x)\,dx$

Practical Components

Do the following questions:
Organise your solutions neatly in your exercise book.

Chapter 12 of Maths Quest 12 Mathematical Methods (pdf – Google Classroom)

- 12E: odd numbered questions
- 12F: odd numbered questions
- 12G: even numbered questions

Chapter 6 of Reasoning and Data (pdf – Google Classroom)

- 6a: (Use CAS) 2, 3, 6, 14, 17, 20
- 6b: 2, 3, 4
- 6c: 1, 4, 5
- 6d: 1, 3, 6

Mathspace

Investigation

See next page

Other

Fun fact: The National Institute for Standards and Technology in the United States is currently holding a competition to standardise cryptographic algorithms that are resistant to attacks by quantum computers. The security of many of the commonly used public-key cryptographic algorithms in use today, such as RSA and Diffie-Hellman, are based on the discrete logarithm problem, and are therefore susceptible to quantum computer-specific attacks based on Shor's Algorithm. Quantum-resistant cryptographic algorithms, on the other hand, derive their security from the assumed computational difficulty of solving other problems, such as those in the study of lattices.

Week 11 Investigation

1. A random variable, X, has its frequency curve defined as:

$$f(x) = \begin{cases} \frac{1}{2} e^{-\frac{1}{2}x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

   a. Draw the graph of $f(x)$
   b. Show that $f(x)$ is a probability density function.
   c. Find the probability, correct to 4 decimal places, that X is:
      i. smaller than 3
      ii. greater than 2.5
      iii. greater than 2.5, given that it is smaller than 3.

2. The wingspan of birds of a particular species has normal distribution with mean 50 cm and standard deviation 5 cm.
   a. Find the probability that a randomly selected bird has a wingspan greater than 60 cm.
   b. If the wingspan is measured to nearest cm, find the probability that a randomly selected bird has a wingspan measured as 50 cm.

3. The length of a certain species of fish has a normal distribution with a mean of 30 cm and a standard deviation of 2.5 cm. An angler caught nine fish whose average length was 27 cm. Is this significantly less than the expected value at the 3σ level?