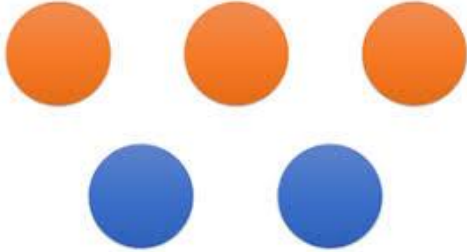


Goals

Goals for this fortnight:

- demonstrate an understanding of the elementary ideas and notation of ratio
- understand the relationship between fractions and ratio
- express a ratio in simplest form
- find the ratio of two quantities
- divide a quantity in a given ratio
- use ratio to describe simple scales
- review identifying common usage of rates such as km/h
- convert between units for rates; for example, km/h to m/s, mL/min to L/h
- complete calculations with rates, including solving problems involving direct proportion in terms of rate
- use rates to make comparisons
- use rates to determine costs; for example, calculating the cost of a tradesman using rates per hour, call-out fees



Theoretical Components

STEP 1

Resources:

PDF file: Week 9 and 10 Notes and Exercises

This Fortnight:

We will be learning:

- Writing and simplifying ratios
- Finding ratios
- Dividing a quantity into a given ratio
- Identify and compare rates
- Using rates

Check this out:

<https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/intro-to-ratios/v/ratios-intro>

Practical Components

STEP 2

Read through Week 9 and 10 Notes and Exercises for instructions on what to do.

There are 5 Exercises in this booklet. Read any worked examples before you begin.

Remember to regularly check Google Classroom for messages.

Portfolio Task

STEP 3

Complete the task at the end of the brief and submit your weekly work. 😊

Other

Remember to check Google Classroom or hawkermaths.com for each week's learning brief



ESSENTIAL MATHEMATICS 2

WEEK 9 AND 10 – RATES AND RATIOS

How are we ever going to use this?

- Mixing paint or other ingredients
- Mixing solutions
- Adjusting bike gears

Both rates and ratios are a comparison of two numbers. A ratio compares two numbers with the *same* unit, for example 20 mL of cordial to 200 mL of water. A rate is a specific type of ratio that compares two numbers with *different* units, for example 60 km per hour.

WRITING AND SIMPLIFYING RATIOS

A ratio is an ordered comparison of two or more quantities. It tells us how much there is of one thing compared to another. They are often used in everyday life, whether it's for dividing up money, mixing paint colours, cooking, or mixing cement.

The order that the words are written in the question corresponds to the order of the values in the ratio, so we need to be careful not to jumble them around.

Example:

Lara mixed 12 mL of white paint with 8 mL of red paint to make dark pink paint.

- a. What is the ratio of red to white paint in the mixture?
- b. What fraction of the mixture was red paint?

Solution:

- a The order is important. In the ratio of red : white, we need to write the quantity of red first.

$$\text{red : white} = 8 : 12$$

- b There is 20 mL of resulting paint and 8 mL came from red paint. That's 8 mL out of 20 mL.

$$\text{Fraction of the paint that was red paint} = \frac{8}{20}$$

Just like we can simplify fractions by multiplying or dividing both the numerator and the denominator by the same amount, we can simplify ratios by multiplying and dividing both numbers in the ratio by the same number.

Example:

Lara's red and white paint mixture is in a ration of 8 : 12. Simplify the ratio 8 : 12.

Solution:

We can divide 8 and 12 by 4.

$$8 : 12 = 2 : 3$$

$$8 \div 4 = 2 \text{ and } 12 \div 4 = 3.$$

Write the answer.

When Lara mixes 8 mL of red with 12 mL of white paint, she is mixing the paint in a ratio of 2 : 3.

We have to be careful with units. All quantities must be in the **same** unit before we can write the quantities in a ratio.

Exercise 1

1. Simplify each ratio.

a. 4 : 10

b. 18 : 12

c. 100 : 20

d. 36 : 20

e. 70 : 7000

f. 20 : 1000

2. Express the following quantities as ratios. Remember to make the units the same and simplify your answer if you can.

a. 40 mL : 1 L

b. 2 L : 500 mL

c. 3 m : 60 cm

d. 5 kg : 500 g

e. 2 hrs: 40 mins

f. 45 mins : $\frac{1}{2}$ hr

3. Lara mixed 6 mL of green paint with 2 mL of blue to make a blue-green paint.
 - a. What is the ratio of green : blue paint in the blue-green mixture?

 - b. Simplify your answer to part a.

 - c. What fraction of the blue-green mixture came from the green paint?

4. The number of students enrolled at a school is 1200. Of these, 625 are male.
 - a. What is the ratio of the number of male students to the number of female students?

 - b. What fraction of the school population is male?

5. A batch of concrete is made from 10 kg of sand, 2 kg of cement, 5 kg of water and 2 kg of gravel.
 - a. Express the parts as a ratio.

 - b. Express each ingredient as a fraction of the whole.

6. A mixture contains 6 parts of oil, 2 parts of insecticide and 10 parts of water by volume. Express the ratio of oil : insecticide : water in simplest form.

FINDING RATIOS

A proportion is a statement that two ratios are equal. To calculate the unknown multiply or divide as required.

Example:

Solve the equation $x : 8 = 9 : 2$

Solution:

To turn 2 into 8, we multiply by 4. We can multiply the ratio $9 : 2$ by 4 to get the ratio $x : 8$.

$$\begin{aligned}x : 8 &= (9 : 2) \times 4 \\x : 8 &= 36 : 8 \\x &= 36\end{aligned}$$

Write the answer.

The solution to the equation $x : 8 = 9 : 2$ is $x = 36$.

Exercise 2

1. Determine the value of the unknown in each equation.

a. $\underline{\quad} : 20 = 1 : 5$

b. $\underline{\quad} : 30 = 1 : 3$

c. $\underline{\quad} : 15 = 2 : 5$

2. The ratio of kilograms to pounds is 1 : 2.2. How many pounds are equal to 20 kg?

- The ratio of miles to kilometres is 1 : 1.6. If Charlie walks 500 miles, how many kilometres has he travelled?

- Kara wants to make an orange-yellow by mixing yellow and orange in the ratio of 3 : 1. How much yellow should she add to 5 mL of orange paint?

- The ratio of sultanas to nuts in a brand of trail mix is always 32 : 56. If a snack pack has 12 sultanas, how many nuts are there?

- For a 2-stroke lawn mower, fuel and oil needs to be premixed in the ratio 50 : 1. If Bob wants to use 1 litre of fuel, how much oil will he need to add to the mixture?

DIVIDING A QUANTITY INTO A GIVEN RATIO

Often, we are given a total amount that needs to be divided into a given ratio.

We use a method called the unitary method to work out how much to allocate to each part of the ratio.

The Unitary Method

- Calculate the **total number of parts** (by adding all the numbers in the ratio).
- Calculate what **one part is worth** (by dividing the given value by the total number of parts)
- Calculate what **each share of the ratio is worth** (by multiplying what one part is worth with each number in the ratio)

Example:

Amir and Keira shared \$720 in the ratio 4 : 5. How much did each person get?

Solution:

Think: There are $4 + 5 = 9$ parts in total, so we can find one part by dividing \$720 by 9 parts to get \$80. We can now use the knowledge that Amir gets 4 parts and Keira gets 5 parts to find each share of the money.

Do:

$$\begin{aligned}\text{Amir's share} &= 4 \times \$80 \\ &= \$320\end{aligned}$$

$$\begin{aligned}\text{Keira's share} &= 5 \times \$80 \\ &= \$400\end{aligned}$$

Check: The total of Amir's share and Keira's share should sum to the total amount:

$$\$320 + \$400 = \$720$$

Exercise 3

1. Find the total number of parts in the following ratios:

a. 2 : 3

b. 7 : 17

c. 5 : 7 : 9

2. Tricia and Luigi invest money into a business in the ratio 4 : 9.
 - a. How many parts has Tricia contributed?

 - b. If Tricia has invested \$2400, how much money does 1 part represent?

 - c. How much has Luigi invested?

3. The number of red, blue, green and white jelly beans in a sweet bag are found to be in the ratio of 4 : 2 : 5 : 2. There are 65 jelly beans in the sweet bag. What is the number of red jelly beans?

4. Cordial is to be mixed in the ratio 1 part cordial concentrate to 9 parts water. If Sandra wants to make 5 Litres of mixed cordial, how much concentrate will she need?

IDENTIFY AND COMPARE RATES

A rate is a ratio between two quantities that are measured in different units. For example, the rate a tap leaks may be 30 mL every 5 minutes. Rates are often expressed as unitary rates where the second quantity in the rate has a measure of 1. The unitary rate for the leaking tap would be 6 mL for every 1 minute, or 6 mL/min.

Common examples of rates include:

- Speeds, eg. 60 km/h, 20 m/s, 400 m every 5 minutes
- Growth rates, eg 20 cm/year, 15 mm/month, 2 kg per week
- Cost of groceries, eg. \$5 per kg, \$1.20 per 100 g, \$10 per box

Handy Conversions

1 metre = 100 centimetres

1 metre = 1000 millimetres

1 kilometre = 1000 metres

1 litre = 1000 millilitres

1 hour = 60 minutes

1 minute = 60 seconds

Converting rates allows us to compare rates given in different units and to also obtain a rate in units suitable for a particular application.

Common applications are comparing unit prices to find the best deal and comparing speeds of different objects.

Example:

Holly runs a 42 kilometre marathon in 3 hours and 30 minutes.

- Find Holly's simplified running rate (speed) in kilometres per hour.
- Convert her speed to metres per second.

Solution:

(a) Find Holly's simplified running rate (speed) in kilometres per hour.

Think: To obtain a rate in km/h, we need to divide the distance in kilometres by the time in hours. (3 hours and 30 minutes = 3.5 hours)

Do:

$$\begin{aligned}\text{Speed} &= \frac{42 \text{ km}}{3.5 \text{ h}} && \text{Divide distance in kilometres by time in hours} \\ &= 12 \text{ km/h} && \text{Simplify the fraction and don't forget units}\end{aligned}$$

(b) Convert her speed to metres per second.

Think: Both the distance unit and time unit are being converted. Let's first change the distance unit. In this case, we are converting distance from kilometres to metres. The number of metres travelled in a given amount of time is 1 000 times greater than the number of kilometres, so we want to multiply the rate by 1 000.

Then we want to convert the new rate in m/h to m/s. There are 60 seconds in a minute, and then 60 minutes in an hour, so the number of metres travelled in a second will be the rate in m/h divided by 60^2 .

Do:

$$\begin{aligned}\text{Speed} &= 12 \text{ km/h} && \text{Write the given rate including units} \\ &= 12 \times 1\,000 \text{ m/h} && \text{Convert the kilometres to metres} \\ &= 12\,000 \text{ m/h} && \text{Simplify the rate} \\ &= \frac{12\,000}{60^2} \text{ m/s} && \text{Convert the rate from per hours to per second} \\ &= 3\frac{1}{3} \text{ m/s} && \text{Simplify the rate}\end{aligned}$$

Exercise 4

1. Convert the following measurements:

a. 20 km to metres

b. $\frac{1}{2}$ Litre to millilitres

c. 40 minutes to seconds

2. Convert the following rates:

a. 50 km/h to m/h

b. 50 km/h to m/s

c. 40 m/s to m/h

d. 40 m/s to km/h

3. An athlete runs 270 metres in 27 seconds. What is his speed in km/h?

4. Use the conversion 1 mile = 1.6093 kilometres to answer the following questions.
- a. If Usain Bolt can run 27.8 miles per hour (mph), what is his speed in km/h?

 - b. The speed limit on a particular road in the US is 70 mph. An Australian visiting the country is more familiar with the speeds using the unit km/h.
 - i. What is the speed limit in kilometres per hour?

 - ii. If a car is travelling at 125 km/h, by how many km/h is the car exceeding the speed limit?
5. A supermarket sells two different brands of eggs. Happy Hens and Sunny Side Up. Happy Hens eggs cost \$6.00 for 12 eggs and Sunny Side Up eggs cost \$1.50 for 5 eggs.
- a. What is the cost in cents per egg from Happy Hens?

 - b. What is the cost in cents per egg from Sunny Side Up?

 - c. Which brand sells eggs at a cheaper price?

USING RATES

We have seen how we can identify and compare rates to decide on the best value for money when comparing similar items or comparing the speed of different objects. As rates arise frequently in everyday life, there are many more practical uses for rates and calculations involving rates.

For example, we can use a pay rate (dollars per hour) to calculate how much money is earned in a week; we can use the speed (km/h) to predict how long it would take to reach a given location; and we can use rates to calculate and compare the cost of different phone plans with differing charge rates for data (\$/GB) and phone calls (\$/min).

Example:

If Steve earns \$157.05 for working 9 hours over the weekend and Tina earns \$144.00 for working 8 hours.

- Who has the better rate of pay?
- How much will Tina earn in a week where she worked 35 hours at the same pay rate?

Solution:

(a) Who has the better rate of pay?

Think: To work this out, we need a common point of comparison, let's work out how much each person earns per hour.

Do:

$$\text{Steve: } \$157.05 \div 9 \text{ hrs} = \$17.45/\text{hr}$$

$$\text{Tina: } \$144.00 \div 8 \text{ hrs} = \$18.00/\text{hr}$$

So now we can say with certainty that Tina's rate of pay is higher.

(b) How much will Tina earn in a week where she worked 35 hours at the same pay rate?

Think: She will receive \$18 for each hour she works. We need to multiply the pay rate by the number of hours.

Do:

$$\begin{aligned} \text{Weekly pay} &= \$18 \times 35 \\ &= \$630 \end{aligned}$$

Exercise 5

1. Michael and Vincent work at their local supermarket as checkout operators. They both work 7 hours every week. Michael earns \$119 per week and Vincent earns \$161 per week.
 - a. Who has the better rate of pay?
 - b. How much more money does Vincent earn than Michael each hour?
2. Dave compares the fuel consumption rates of two similar 2018 model Holden Commodores. One has a 4 cylinder engine while the other has a 6 cylinder engine. Both are automatic sedans with the same fuel capacity of 61 litres.
 - a. If the price of fuel is \$1.229 per litre, what is the cost of completely filling the tank for each car? *Give your answer in dollars. Round your answer to the nearest cent.*
 - b. The fuel consumption rate is 7.8 L per 100km for the 4 cylinder, and is 8.8 L per 100km for the 6 cylinder. Work out how far each car can travel on a full tank of fuel. Round your answer to the nearest kilometre.

3. A fitness club charges a membership fee of \$90 plus \$22 per week. How much would you pay if you went to the fitness club for 11 weeks?

4. Zuber is a taxi services that charges \$1.80 pick-up fee and \$1.25 per kilometre of travel. What is the total charge for a 10km journey?

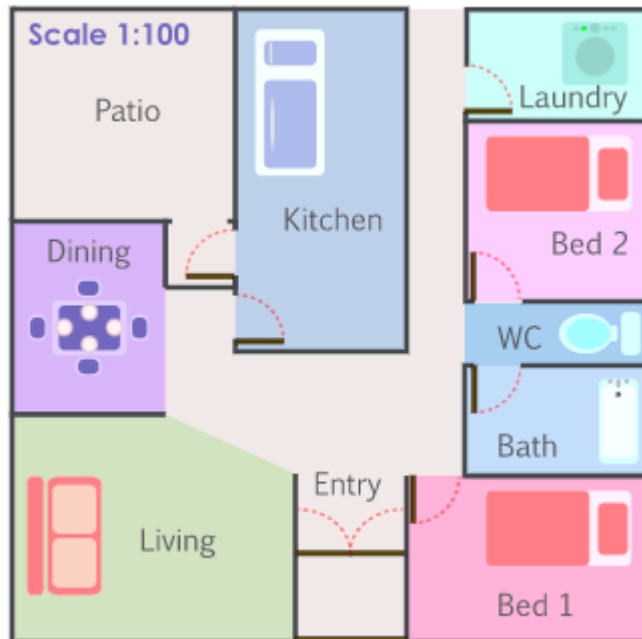
5. Luke is a locksmith. On weekends, if he is on call, he gets paid \$64.50 per hour plus a flat rate call out fee of \$75. On Saturday he did 3 jobs that took 1.5, 0.25 and 1.25 hours respectively to complete. How much did he earn on Saturday?

6. If three kids can eat three hot dogs in three minutes, how many minutes would it take 100 kids to eat 100 hot dogs?

Week 9/10 Portfolio Task

Use the diagram to answer the following questions.

- a. According to the scale of the diagram, 1 cm on the diagram represents how many metres in the house?
- b. Using a ruler, Neville measures the length of the kitchen on the plan and finds it to be 6 cm. How many metres does this represent?



- c. Using a ruler, Neville measures the width of the kitchen on the plan and finds it to be 3 cm. How many metres does this represent?
- d. Neville wants to tile the floor of the kitchen. If each tile is 50 cm by 50 cm, how many tiles will Neville need?
- e. It costs \$45 for a box of 8 tiles.
 - i. How many boxes will Neville have to buy?
 - ii. How much will the tiles cost to purchase?
- f. A tiler charges \$60 per square metre to lay tiles. How much will Neville have to pay the tiler to lay the tiles?
- g. How much will it cost in total to tile the kitchen floor?

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher.	2	3		/6
Portfolio Task	Student completes the portfolio task of the week to an acceptable standard set by the teacher.	2	2		/4
Reasoning and Communications	Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown.	4	-		/4
Concepts and Techniques	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae	4	-		/4
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details.	2	-		/2
		FINAL			/20

Student Reflection: 😊

How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?