2017 SMM1 Week 1 Investigation

Part A: Prime factorisation

View the following website and investigate prime factors, greatest common factor and lowest common multiple.

http://www.mathplayground.com/factortrees.html

Answer the following, showing working:
What is the prime factorisation of 84?
What is the prime factorisation of 72?
What are the GFC and the LCM of 84 and 72?

Part B: Proof that $\sqrt{2}$ is irrational

How do we know that square root of 2 is an irrational number? In other words, how do we know that √2 wouldn't have a pattern in the decimal sequence? Maybe the pattern is very well hidden and is really long, billions of digits? Even if you check it till million first digits, maybe the pattern is just longer than you were able to print the digits out with your computer?
Here is where mathematical proof comes in. It does not rely on computers at all, but instead it is a "proof by contradiction"—if √2 WERE a rational number, then we'd get a contradiction.

Proof

Let us suppose that $\sqrt{2}$ is rational and, therefore, can be written as a fraction in the form $\frac{p}{q}$ where p and q are positive integers *with no common factors* (if there were any common factors they would be cancelled). So

$$\sqrt{2} = \frac{p}{q}$$

$$\text{then } 2 = \frac{p^2}{q^2} \text{ (squaring both sides)}$$

$$\text{which gives } 2q^2 = p^2$$

This implies that p^2 must be even as $2q^2$ is even (any integer multiplied by 2 is even) and thus p is even. Remember an even number squared is even. Therefore p can be expressed in the form $2k$ for some integer k.

$$\text{Thus } 2q^2 = (2k)^2$$

$$2q^2 = 4k^2$$

$$q^2 = 2k^2$$

As $2k^2$ must be even then q^2 must be even and thus q must be even. But p and q were said to have no common factor, hence a contradiction exists. So our original assumption was wrong.

Therefore, p and q cannot be found so that $\sqrt{2} = \frac{p}{q}$. Hence, $\sqrt{2}$ must be irrational.

Use the above method to prove that √5 is an irrational number. It is important to be aware that there are other ways to prove that √2 (and √5) are irrational. You may like research and present an alternative proof.

Part C

A study was made of 200 students to determine what TV shows they watched.

- 22 students do not watch any TV shows
- 73 students watch only Pretty Little Liars
- 136 students watch Pretty Little Liars
- 14 students watch only Vampire Diaries and Gossip Girl
- 31 students watch only Pretty Little Liars and Gossip Girl
- 63 students watch Vampire Diaries
- 135 students do not watch Gossip Girl

Construct a Venn diagram and use this to show the number of students who watch all three programs. Show appropriate working.