CALCULUS

INTEGRALS

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

where f is continuous on $[a, b]$and $F' = f$

FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

where f is continuous on $[a, b]$ and $F' = f$

INTEGRATION PROPERTIES

$$\int_a^b cf(x)dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b f(x)dx = 0 \text{ and } \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)\,dx$$

APPROXIMATING DEFINITE INTEGRALS

Left-hand and right-hand rectangle approximation

$$L_n = \Delta x\sum_{k=0}^{n-1} f(x_k) \qquad R_n = \Delta x\sum_{k=0}^{n} f(x_k)$$

Midpoint rule

$$M_n = \Delta x\sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

Trapezoid rule

$$T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n))$$

COMMON INTEGRALS

$$\int k\ dx = kx + C$$

$$\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int x^{-1}\ dx = \int \frac{1}{x}dx = \ln|x| + C$$

$$\int \frac{1}{ax+b}\ dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int \ln x\ dx = x\ln(x) - x + C$$

$$\int e^x\ dx = e^x + C$$

$$\int \cos x\ dx = \sin x + C$$

$$\int \sin x\ dx = \cos x + C$$

$$\int \sec^2 x\ dx = \tan x + C$$

$$\int \sec x\tan x\ dx = \sec x + C$$

$$\int \csc x\cot x\ dx = -\csc x + C$$

$$\int \csc^2 x\ dx = -\cot x + C$$

$$\int \tan x\ dx = \ln|\sec x| + C$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{a^2+u^2}\ dx = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\ dx = \sin^{-1}\left(\frac{u}{a}\right) + C$$

TRIG SUBSTITUTIONS

EXPRESSION	SUBSTITUTION	EXPRESSION EVALUATION	IDENTITY USED
$\sqrt{a^2-x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\ d\theta$	$\sqrt{a^2 - a^2\sin^2\theta}$ $= a\cos\theta$	$1 - \sin^2\theta$ $= \cos^2\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\ d\theta$	$\sqrt{a^2\sec^2\theta - a^2}$ $= a\tan\theta$	$\sec^2\theta - 1$ $= \tan^2\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\ d\theta$	$\sqrt{a^2 + a^2\tan^2\theta}$ $= a\sec\theta$	$1 + \tan^2\theta$ $= \sec^2\theta$

L'HOPITAL'S RULE

$$\text{if } \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty} \text{ then } \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a} f'(x)/g'(x)$$

INTEGRATION BY PARTS

$$\int u\ dv = uv - \int v\ du \quad \text{where} \quad v = \int dv \text{ or } \int f(x)g'(x)\ dx = f(x)g(x) - \int f'(x)g(x)dx$$

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MATHS REFERENCE SHEET COLLECTION

A reference sheet for the hawkermaths.com senior maths program

Mathematical Methods
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INTEGRATION BY SUBSTITUTION

$$\int_a^b f(g(x))\ g'(x)dx = \int_{g(a)}^{g(b)} f(u)\,du$$

where $u = g(x)$and $du = g'(x)dx$