

2014 SM4 Revision Questions – Distributions

Normal

Q1. Professor Halen has 184 students in his college mathematics class. The scores on the semester exam are normally distributed with a mean of 72.3 and a standard deviation of 8.9. How many students in the class can be expected to receive a score between 82 and 90? Express answer to the *nearest student*.

Q2. X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

a) $P(x < 40)$

b) $P(x > 21)$

c) $P(30 < x < 35)$

Q3. A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr.

a) What is the probability that a car picked at random is travelling at more than 100 km/hr?

b) What proportion of cars are travelling at over 100 km/hr given that they are travelling over 90 km/hr?

Q4. Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

Q5. A machine is used to fill soda bottles. The amount of soda dispensed into each bottle varies slightly. Suppose the amount of soda dispensed into the bottles is normally distributed. If at least 99% of the bottles must have between 585 and 595 millilitres of soda, find the greatest standard deviation, to the *nearest hundredth*, that can be allowed.

Q6. A shoe manufacturer collected data regarding men's shoe sizes and found that the distribution of sizes exactly fits the normal curve. If the mean shoe size is 11 and the standard deviation is 1.5, find:

a) the probability that a man's shoe size is greater than or equal to 12.5.

b) the probability that a man's shoe size is greater than or equal to 12.5 given that it is greater than or equal to 8?

Q7. Use the normal distribution to approximate the binomial distribution and find the probability of getting 15 to 18 heads out of 25 flips. Compare this to what you get when you calculate the probability using the binomial distribution. Note: As the normal curve assumes continuous data and flipping a coin is discrete you need to find the area between 14.5 and 18.5.

Q8. True/false: For any normal distribution, the mean, median, and mode will have the same value.

Q9. True/false: In a normal distribution, 11.5% of scores are greater than $Z = 1.2$.

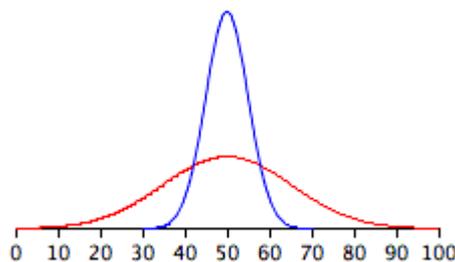
Q10. True/false: The percentile rank for the mean is 50% for any normal distribution.

Q11. True/false: A Z-score represents the number of standard deviations above or below the mean.

Q12. True/false: The standard deviation of the blue distribution shown below is about 10.

Q13. True/false: In the figure below, the red distribution has a larger standard deviation than the blue distribution.

Q14. True/false: The red distribution has more area underneath the curve than the blue distribution does.



Binomial

Q1. Ninety percent of the trees planted by a landscaping firm survive. What is the probability that of the next 13 trees planted:

- a) at most ten will survive?
- b) at least ten will survive?
- c) exactly ten will survive?

Q2. A machine produces parts of which 0.5% are defective. If a random sample of ten parts produced by this machine contains more than one defective part, the machine is shut down for repairs. Find the probability that the machine will be shut down for repairs based on this sampling plan.

Q3. Suppose that you take a five-question multiple-choice quiz and guess the answers. Each question has possible answers a, b, c, d and only one is correct.

- a) what is the probability that you guess more than half of the answer correctly?
- b) what is the probability of guessing the answer to the first question correctly?

Q4. There are 10 red and 20 blue balls in a box. A ball is chosen at random and it is noted whether it is red. The process is repeated, returning the ball 10 times. Calculate the expected value and the standard deviation of this game.

Q5. An agent sells life insurance policies to five equally aged, healthy people. According to recent data, the probability of a person living in these conditions for 30 years or more is $\frac{2}{3}$. Calculate the probability that after 30 years:

- a) all five people are living,
- b) at least three people are still living,
- c) less than two people are still living.

Q6. Eight friends take a picnic to a cricket match. As her contribution to the picnic, Hilda buys eight sandwiches at a supermarket. She selects the sandwiches at random from those on display. The probability that a sandwich is suitable for vegetarians is independently 0.3 for each sandwich.

(a) Find the probability that, of the eight sandwiches, the number suitable for vegetarians is:

- (i) 2 or fewer;
- (ii) exactly 2;
- (iii) more than 3.

(b) Two of the eight friends are vegetarians. Hilda decides to ensure that the eight sandwiches she takes to the match will include at least two suitable for vegetarians. If, having selected eight sandwiches at random, she finds they include fewer than two suitable for vegetarians she will replace one, or if necessary two, of the sandwiches unsuitable for vegetarians with the appropriate number of sandwiches suitable for vegetarians.

State whether or not the binomial distribution provides an appropriate model for the number of sandwiches suitable for vegetarians which Hilda takes to the match. Explain your answer.

Q7. A pharmaceutical lab states that a drug causes negative side effects in 3 out of every 100 patients. To confirm this affirmation, another laboratory chooses 5 people at random who have consumed the drug. What is the probability of the following events?

- a) none of the five patients experience side effects.
- b) at least two experience side effects
- c) what is the average number of patients that the laboratory should expect to experience side effects if they choose 100 patients at random?

Poisson

Q1. The number of traffic accidents that occurs on a particular stretch of road during a month follows a Poisson distribution with a mean of 9.4. Find the probability that less than two accidents will occur on this stretch of road during a randomly selected month.

Q2. The number of road construction projects that take place at any one time in a certain city follows a Poisson distribution with a mean of 3. Find the probability that exactly five road construction projects are currently taking place in this city.

Q3. The university police department must write, on average, five tickets per day to keep department revenues at budgeted levels. Suppose the number of tickets written per day follows a Poisson distribution with a mean of 8.8 tickets per day. Find the probability that less than six tickets are written on a randomly selected day from this distribution.

Q4. Vehicles pass through a junction on a busy road at an average rate of 300 per hour.

- a. Find the probability that none passes in a given minute.
- b. What is the expected number passing in two minutes?
- c. Find the probability that this expected number actually pass through in a given two-minute period.
- d.

Q5. A company makes electric motors. The probability an electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors?

Q6. Twenty sheets of aluminum alloy were examined for surface flaws. The frequency of the number of sheets with a given number of flaws per sheet was as follows:

Number of flaws	Frequency
0	4
1	3
2	5
3	2
4	4
5	1
6	1

What is the probability of finding a sheet chosen at random which contains 3 or more surface flaws?

Answers

Normal Distribution

Q1. 21 students

Q2. a) 0.9938 b) 0.9878 c) 0.3944

Q3. a) 0.1587 b) 0.3174

Q4. Tom's score was higher than 80.23% of students, thus **yes**.

Q5. 1.95

Q6. a) 0.1587 b) 0.1624

Q7. 0.2037 (binomial 0.2049)

Q8. T

Q9. T

Q10. T

Q11. T

Q12. F

Q13. T

Q14. F

Binomial Distribution

Q1. a) 0.1339 b) 0.9658 c) 0.0349

Q2. 0.0011

Q3. 0.1035 b) 0.25

Q4. a) 20 b) 2.58

Q5. 0.1317 b) 0.7901 c) 0.2099

Q6. a) (i) 0.5518 (ii) 0.2965 (iii) 0.1941

b) No, independence requirement not met, probability of success each trial not the same.

Q7. a) 1 b) 0.0085 c) 3

Poisson

Q1. 0.0009

Q2. 0.1008

Q3. 0.1284

Q4. a) 0.0067 b) 10 c) 0.1251

Q5. 0.1008

Q6. 0.4040