GENERAL MATHEMATICS 2

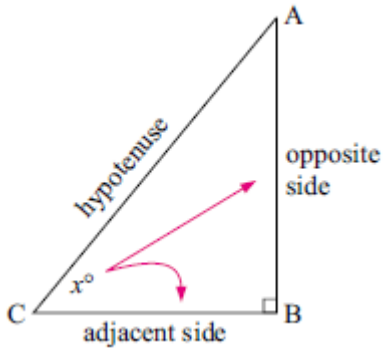
WEEK 14 NOTES & EXERCISES – TERM 4

NAMING THE SIDES OF RIGHT-ANGLED TRIANGLES

In a right-angled triangle, the side opposite the right angle is called the *hypotenuse.*

The other two sides are named in relation to the angle in question, $x°$ (other letters can be used).

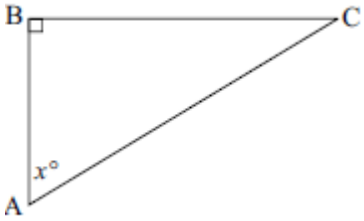
The side furthest away from the angle is called the *opposite* side. The remaining side, which is next to the angle, is called the *adjacent* side.



Exercise Set 1.

Q1. Label the sides of the following triangles (hypotenuse, opposite, adjacent).

a)



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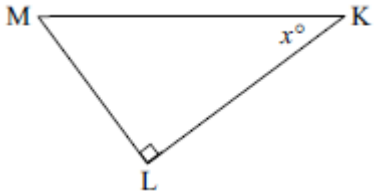

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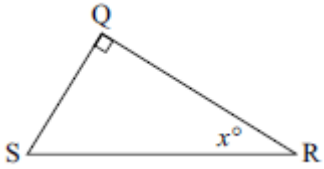

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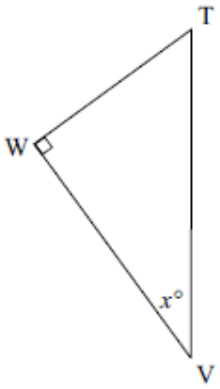

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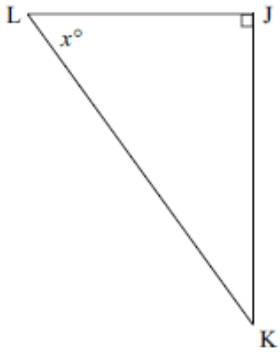
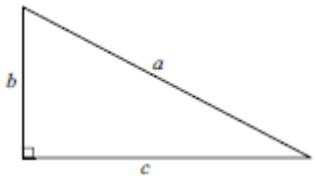


PYTHAGORAS' THEOREM

In a right-angled triangle, the sum of the squares of the the two shorter sides is equal to the square of the hypotenuse.

Remember: A right-angle is an angle of 90°. The hypotenuse is the side opposite the right-angle and is always the longest side.

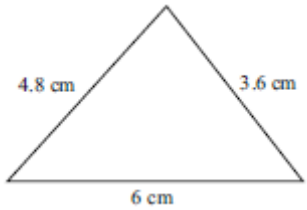


$$a^2 = b^2 + c^2$$

Thus, if we know the lengths of two sides we can calculate the length of the third side. We can also determine if a triangle is a right-angled triangle.

EXAMPLE

(i) Determine if the following triangle is right-angled and if so sketch and mark the right angle.

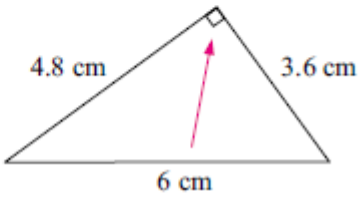


$$\begin{aligned} \text{Longest side, squared} &= 6^2 = 36 \\ \text{Sum the squares of the other two sides} &= 4.8^2 + 3.6^2 \\ &= 23.04 + 12.96 \\ &= 36 \end{aligned}$$

$$\text{i.e.,} \quad 6^2 = 4.8^2 + 3.6^2$$

i.e., Pythagoras' theorem is true for this triangle, hence it is right-angled.

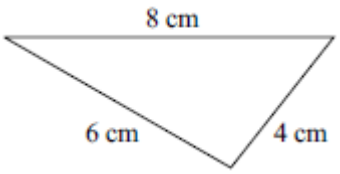
The right-angle is opposite the longest side, hence



Exercise Set 2

Q1. Determine which of these triangles are right-angled and if so mark the angle accordingly.

a)



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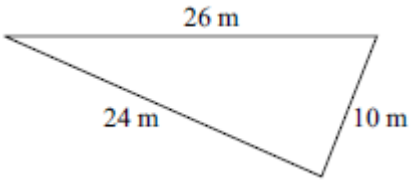

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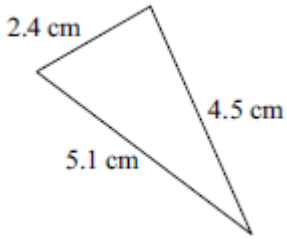
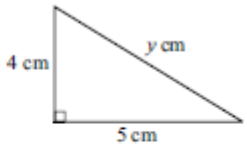


EXAMPLE 1

Find the length of the hypotenuse in the given triangle.

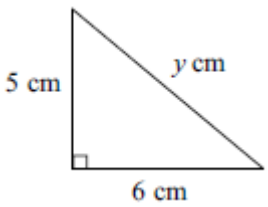


$$y^2 = 4^2 + 5^2$$
$$\therefore \quad y^2 = 16 + 25$$
$$\therefore \quad y^2 = 41$$
$$\therefore \quad y = \sqrt{41} = 6.4 \quad \text{(to 1 d.p.)}$$

i.e., the length of the hypotenuse is 6.4 cm.

Q2. Find the length of the hypotenuse, correct to one decimal place.

a)



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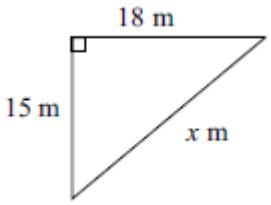

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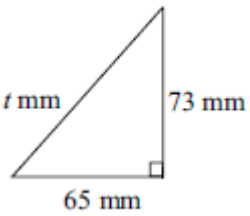

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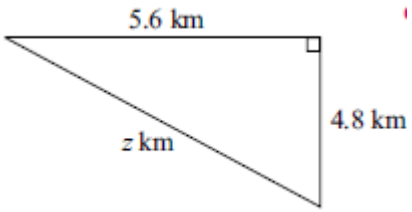

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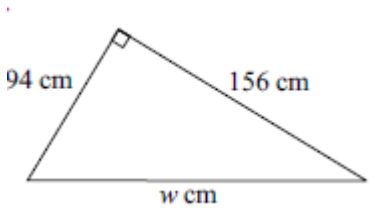
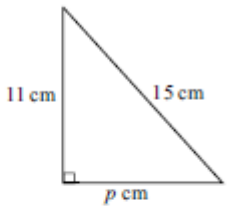


EXAMPLE 2

Find the length of the unknown side in the given triangle.



Using Pythagoras' theorem in this triangle

$$p^2 + 11^2 = 15^2$$

i.e., $p^2 + 121 = 225$

$\therefore \quad p^2 = 225 - 121$

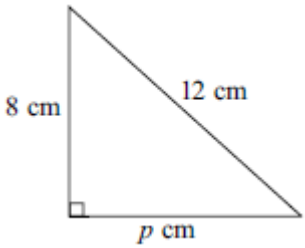
$\therefore \quad p^2 = 104$

$\therefore \quad p = \sqrt{104} = 10.2$ (to 1 d.p.)

i.e., the length of the unknown side is 10.2 cm.

Q3. Find the length of the unknown side.

a)



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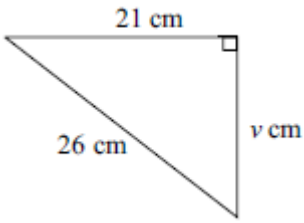

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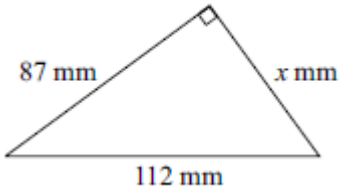


Q4. A ship leaves Sydney Heads and travels 45 km due East. It then turns and travels 86 km due North. How far is it from its starting point? Draw a picture to help answer this question.

DEFINING THE TRIGONOMETRIC RATIOS

Trigonometry involves solving problems using the ratio of pairs of sides in right-angled triangles.

Remember the sides are called *hypotenuse, opposite* and *adjacent.*

The ratio $\frac{opposite}{hypotenuse}$ is called the **sin** ratio. (sin is short for sine)

The ratio $\frac{adjacent}{hypotenuse}$ is called the **cos** ratio. (cos is short for cosine)

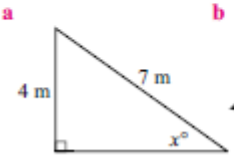
The ratio $\frac{opposite}{adjacent}$ is called the **tan** ratio. (tan is short for tangent)

EXAMPLE 1

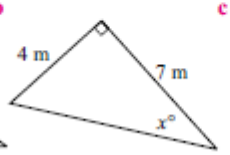
For the following triangles, state whether $\frac{4}{7} =$

A $\sin x$ **B** $\cos x$ **C** $\tan x$ **D** none of these ratios

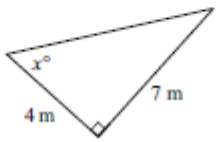
a



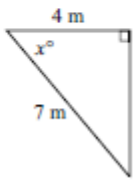
b



c



d



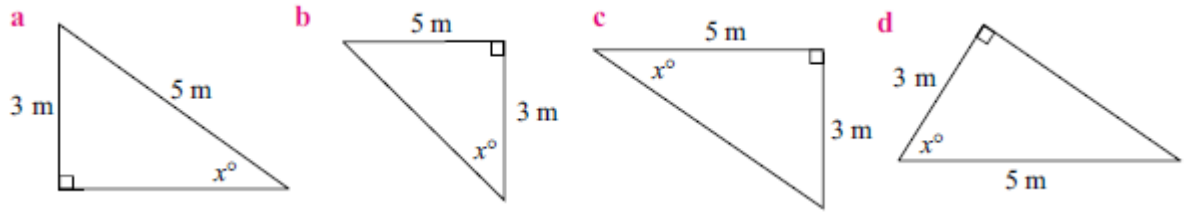
a $\frac{4}{7} = \frac{\text{opposite}}{\text{hypotenuse}} = \sin x°$ **b** $\frac{4}{7} = \frac{\text{opposite}}{\text{adjacent}} = \tan x°$

c $\frac{4}{7} = \frac{\text{adjacent}}{\text{opposite}}$ =none of these **d** $\frac{4}{7} = \frac{\text{adjacent}}{\text{hypotenuse}} = \cos x°$

Exercise Set 3.

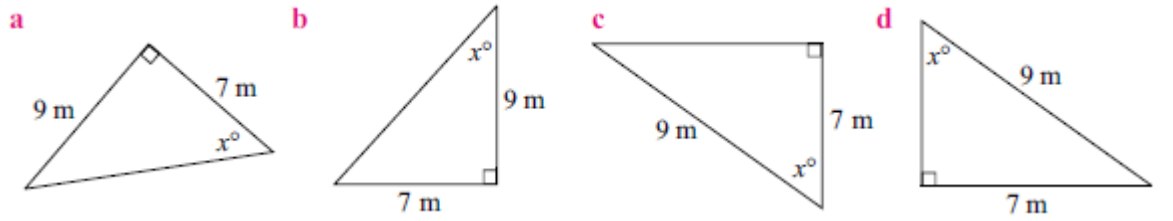
Q1. For the following triangles, state whether $\frac{3}{5}$ =

A $\sin x$ **B** $\cos x$ **C** $\tan x$ **D** none of these ratios



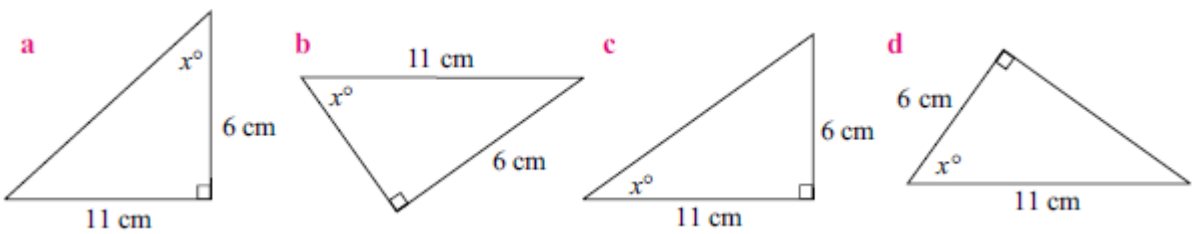
Q2. For the following triangles, state whether $\frac{7}{9}$ =

A $\sin x$ **B** $\cos x$ **C** $\tan x$ **D** none of these ratios



Q3. For the following triangles, state whether $\frac{6}{11}$ =

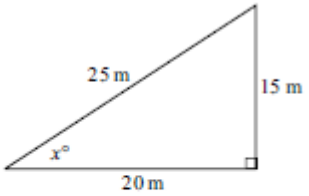
A $\sin x$ **B** $\cos x$ **C** $\tan x$ **D** none of these ratios



FINDING VALUES OF TRIGONOMETRIC RATIOS

EXAMPLE 2

Calculate the value of $\sin x^\circ$, $\cos x^\circ$ and $\tan x^\circ$ in the triangle alongside, which is not drawn to scale.



$$\sin x^\circ = \frac{O}{H} = \frac{15}{25} = \frac{3}{5} \text{ or } 0.6$$

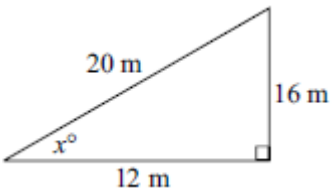
$$\cos x^\circ = \frac{A}{H} = \frac{20}{25} = \frac{4}{5} \text{ or } 0.8$$

$$\tan x^\circ = \frac{O}{A} = \frac{15}{20} = \frac{3}{4} \text{ or } 0.75$$

Exercise Set 4

Q1. Write down the values of sin x°, cos and tan x° in the triangles below.

a)

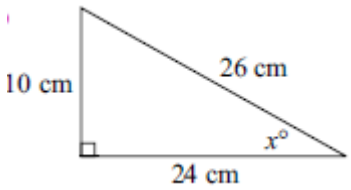


sin x° =

cos x° =

tan x° =

b)

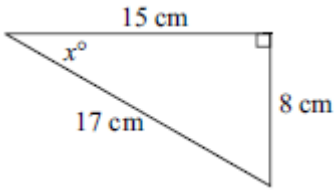


sin x° =

cos x° =

tan x° =

c)

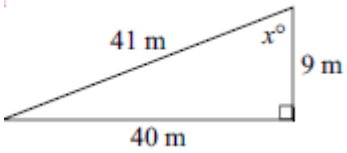


sin x° =

cos x° =

tan x° =

d)

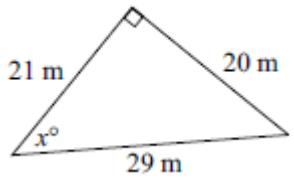


sin x° =

cos x° =

tan x° =

e)

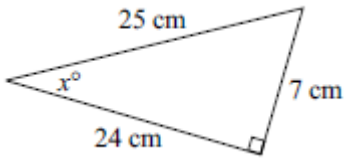


sin $x°$ =

cos $x°$ =

tan $x°$ =

f)



sin $x°$ =

cos $x°$ =

tan $x°$ =